Distributed Dynamic Event-Triggered Control for Leader-Following Multi-Agent Systems

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Abstract. This paper is devoted to address the distributed dynamic event-triggered consensus problem for leader-following multi-agent systems with general linear dynamics. Compared with the existing event-triggered scheme, the proposed dynamic event-triggered scheme involves internal dynamic variables, which play an important role to reduce the inter-agent communications. Besides, distributed event-triggered protocol is designed, which only uses the information from neighboring agents instead of global information . It is shown that the state of followers converge to the leader, the triggering number of the agents reduce significantly and the feasibility of the proposed control scheme is verified by excluding the Zeno behavior. Finally, a numerical simulation is provided to illustrate the correctness of theoretical results.

Keywords: Dynamic event-triggered control, Leader-following, General linear multi-agent systems, Zeno behavior

1. INTRODUCTION

In recent years, multi-agent systems (MASs) have attracted intensive attention due to the broad applications in various fields, e.g., formation of satellites, unmanned vehicles, multiple robots [1]-[6]. The consensus problem for MASs have received great attention due to its potential benefits when compared with a single individual include better flexibility, adaptability, and performance [7]-[9]. The purpose of consensus is to design control protocols based on local information guaranteeing all agents reach an agreement on a common value. The consensus problem can be divided into the leader-following consensus problem and the leaderless consensus problem.

Great efforts have been made for Leader-following consensus and leaderless consensus in the past decade. The leader-following consensus has been an active area of research. In [10], a leader-following consensus problem was proved that if all the agents were jointly connected with their leader, their states would converge to the leader. In [11], a neighbor-based controller together with a neighbor-based state-estimation rule was proposed to solve the consensus problem for leader-following MASs. In [12], a full state feedback consensus protocol was proposed to solve the leader-following consensus problem for high order linear multi-agent systems. It was found that in many studies, the leader-following system helped save energy [13], and enhance the communication and orientation [14], [15]. A necessary condition for the leader-following consensus is that all the agents are connected with the leader [16].

In the aforementioned works on the consensus problem, continuous information communication is needed constantly, which requires ideal communication environment and abundant communication resource . But it is difficult to guarantee the continuous communication because communication devices are mostly powered by batteries, which have limited energy resource and processing capability. The event-triggered control strategy is developed as an important method to avoid continuous communications in recent years. In event-triggered strategies, inter-agent communications occur only when some specific events are triggered. Event-triggered and self-triggered consensus algorithms are presented for single-integrator agents over undirected connected communication topologies in [17]-[19]. In [20], the event-triggered broadcasting control protocols are proposed for single-integrator and double-integrator MASs. The event-triggered consensus problem for linear MASs has been studied in [21]–[28]. Particularly, several state feedback and observer-based output feedback event-triggered consensus protocols are studied for linear multiagent networks in [29]–[32] and [33]. Furthermore, the leader-following consensus problem of second-order MASs is solved by proposing a distributed event-triggered sampling scheme in [34]. The event-triggered controller is designed to solve the leader-following consensus problem of linear MASs in [35], [36]. A key challenge in event-triggered control for MASs is how to design event-triggered controllers to determine when to communication without any global information.

In [37], a new class of event-trigged mechanisms is proposed by introducing an internal dynamic variable. Inspired by the aforecited works, we propose dynamic event-triggered protocol by involving an internal dynamic variable in this paper, and extend it to leader-following general linear multi-agent systems. The contributions of this paper can be summarized as follows:

- A novel distributed dynamic event-triggered scheme is proposed in this paper, which does not need continuous communication among agents. Each agent sends its information to their neighbours only at its own triggering instants. Compared with static event-triggered scheme, the proposed dynamic event-triggered scheme reduce the communication load of the system by reducing the triggering number.

- The distributed dynamic event-triggered protocol we proposed yield consensus exponentially fast and the results show that they are free from Zeno behavior.

The rest of this paper is arranged as follows. Preliminaries and the problem formulation are introduced in Section II. The main results are stated in Section III. Simulations are illustrated in Section IV. Finally, we conclude this paper in Section V.

Notations: R^n denotes the set of n -dimensional real column vectors. $R^{n\times n}$ represents the set of $m\times n$ dimensional real matrices. I_n denotes the $m\times n$ dimensional identity matrix. For a vector or matrix X, X^T denotes its transpose and $\|X\|$ denotes its Euclidean norm. $\otimes$ denotes the Kronecker product of matrices. The following properties of the Kronecker product are useful in this paper: $(A+B)\otimes C = A\otimes C + B\otimes C$, $C\otimes(A+B)= C\otimes A + C\otimes B$.

2. PRELIMINARIES

2.1. Graph Theory

We describe the communication topology among the agents using an undirected graph. Considering an multi-agent system(MAS) consisting of a group of N Ninteracting agents. $\mathcal{G}(\mathcal{V},\mathcal{E})$ is a weighted undirected graph, where $\mathcal{V}=\{1,2,...,N\}$ is the index set of N agent and $\mathcal{E}\in\mathcal{V}\times\mathcal{V}$ is the edge set of paired agents. An edge $(i,j)\in\mathcal{E}$ means the node i can receive information from node j. $\mathcal{A}=[a_{ij}]\in R^{N\times N}$ is the adjacency matrix with non-negative elements, where $a_{ij}=1 \Leftrightarrow if\,(i,j)\in\mathcal{E}$ and $a_{ij}=0$ otherwise. The Laplacian matrix $L=[l_{ij}]\in R^{N\times N}$ is defined as $l_{ij}=-a_{ij},\ i\neq j$, and $l_{ii}=\sum_{j=1,j\neq i}^{N} a_{ij}$. The leader agent is labeled as 0, $\bar{\mathcal{G}}$ is a graph which consists of the subgraph $\mathcal{G}$ and the leader 0. Let $D=diag(b_1,b_2,...,b_N)$, where $b_i=1$ if agent i can receive information from the leader and $b_i=0$, otherwise.

2.2. Problem Statement

In this paper, we consider a multi-agent system consisting of a leader and N followers. For each $i\in V$, the leader and the i-th follower have the generic linear system dynamics described by

$$\dot{x}_0(t)=Ax_0(t) \tag{1}$$

$$\dot{x}_i(t)=A\dot{x}_i(t)+Bu_i(t) \quad i=1,2,...,N \tag{2}$$

where $x_0(t)\in R^n$ is the state of the leader, $x_i(t)\in R^n$ is the state of the follower i, $u_i(t)\in R^p$ is the control input of the follower i. A and B are known matrices with appropriate dimensions. For this MAS, the pair (A,B) is stabilizable.

Definition 1. Consider the leader-following system (1) , (2), the leader-follower consensus problem can be solved by designing an controller that the following equation holds for any initial condition.

$$\lim_{t\to\infty}\|x_i(t)-x_0(t)\|=0,\quad i=1,2,...N \tag{3}$$

Before proceeding, the following lemmas are needed.

Lemma 1. If the undirected communication graph $\bar{\mathcal{G}}$ is connected, then the matrix $H=L+D$ is positive define.

Lemma 2. (Young's inequality): Given any $x,y\in R$, for any $a>0$,

$$xy<\frac{x^2}{2a}+\frac{ay^2}{2} \tag{4}$$

3. MAIN RESULT

Without loss of generality, we assume that the control input of the leader is supposed to be zero. The communication graph $\bar{\mathcal{G}}$ among the agents is assumed to satisfy the following assumption.

Assumption 1. The subgraph $\mathcal{G}$ associated with the followers is undirected and the graph $\bar{\mathcal{G}}$ contains a directed spanning tree with the leader as the root.

Assumption 2. The pair (A,B) is stabilizable.

In the design of the traditional consensus protocol, continuous communication is needed between inter-agent. In [38], the traditional distributed consensus protocol is proposed as follows:

$$u_i(t)=-K\left[\sum_{j=1}^{N}a_{ij}\left(x_i(t)-x_j(t)\right)+b_i\left(x_i(t)-x_0(t)\right)\right] \tag{5}$$

The traditional distributed consensus protocol may waste communication resources, so event-triggered controller is developed to solve the tracking control problem in this paper.

The state measurement error for each agent is defined as:

$$e_i(t)=\tilde{x}_i(t)-x_i(t),\ i=1,...,N \tag{6}$$

where $\tilde{x}_i(t)=e^{A(t-t_k^i)}x_i(t_k^i),\forall t\in[t_k^i-t_{k+1}^i],i=1,...,N$, and t_k^i is the latest triggering instant and $x_i(t_k^i)$ is the last broadcast state of agent i, $k=1,2,...,N$ and $\tilde{x}_0(t)=x_0(t)$.

For each follower, we propose the following event-triggered control protocol:

$$u_i(t)=-K\left[\sum_{j=1}^{N}a_{ij}\left(\tilde{x}_i(t)-\tilde{x}_j(t)\right)+b_i\left(\tilde{x}_i(t)-x_0(t)\right)\right] \tag{7}$$

Since (A,B) is stabilize, for any $\theta>0$, there exists a positive define matrix $P>0$, and P is the solution of the following Riccati inequality:

$$PA + A^T P - 2\theta PBB^T P + \theta I < 0 \tag{8}$$

where K is designed as $K = B^T P$.

The tracking error is define as $z_i(t) = x_i(t) - x_0(t)$, i=1,…,N, then

$$\begin{aligned}\dot{z}_i(t) &= Az_i(t) \\ &\quad - BK\left[\sum_{j=1}^{N} a_{ij}\left(\tilde{x}_i(t) - \tilde{x}_j(t)\right) + b_i\left(\tilde{x}_i(t) - x_0(t)\right)\right] \\ &= Az_i(t) \\ &\quad - BK\left[\begin{aligned}&\sum_{j=1}^{N} a_{ij}\left(z_i(t) - z_j(t) + e_i(t) - e_j(t)\right) \\ &+ b_i\left(z_i(t) + e_i(t)\right)\end{aligned}\right]\end{aligned} \tag{9}$$

Firstly, the distributed static event-triggered condition for each agent i is given by

$$\|e_i(t)\| \le \sqrt{\frac{\sigma_i \lambda_1}{2\gamma\|\Gamma\| + 2\lambda_1}}\|\tilde{z}_i(t)\| \tag{10}$$

where $\gamma = \sum_{i=1}^{N}\left[\sum_{j=1}^{N} 4a_{ij} + 4b_i\right]$, $\Gamma = PBB^T P$,

$0 < \sigma_i < \min\left\{1, \frac{2\gamma\|\Gamma\| + 2\lambda_1}{\lambda_1}\right\}$,

$\tilde{z}_i(t) = \sum_{j=1}^{N} a_{ij}\left(\tilde{x}_i(t) - \tilde{x}_j(t)\right) + b_i\left(\tilde{x}_i(t) - x_0(t)\right)$,

Remark 1. We call (10) static event-triggered scheme since it does not involve any dynamic variables except $x_i(t)$ and $x_j(t)$. The main purpose of using the dynamic event-triggered control is to reduce the overall need of actuation updates and communication between agents, so in the following we propose a dynamic event-triggered control protocol.

We propose to enrich the static event-triggered scheme with an internal dynamic variable η satisfying the following differential equation:

$$\begin{aligned}\dot{\eta}_i(t) &= -\beta_i \eta_i(t) \\ &\quad + \xi_i\left[\sigma_i \lambda_1 \|\tilde{z}_i(t)\|^2 - (4\gamma\|\Gamma\| + 2\lambda_1)e_i^2(t)\right]\end{aligned} \tag{11}$$

where $\tilde{z}_i(t) = \sum_{j=1}^{N} a_{ij}\left(\tilde{x}_i(t) - \tilde{x}_j(t)\right) + b_i\left(\tilde{x}_i(t) - x_0(t)\right)$ and

$\eta_i(0) > 0,\ \beta_i > 0,\ \xi_i \in [0,1],\ \sigma_i \in [0,1)$.

Then the distributed dynamic event-triggered condition for each agent i is given by

$$\|e_i(t)\| \le \sqrt{\frac{\frac{\eta_i(t)}{\theta_i} + \sigma_i \lambda_1 \|\tilde{z}_i(t)\|^2}{4\gamma\|\Gamma\| + 2\lambda_1}} \tag{12}$$

Theorem 1. Consider the MAS (1), (2) with the event-triggered controller (7). Suppose that Assumption 1 and Assumption 2 hold. Choose $K = B^T P$ and $\Gamma = PBB^T P$, where $P > 0$ is defined as in (8). Select $\theta_i > \frac{1-\xi_i}{\beta_i}$, $\eta_i(t)$ is defined as in (11). Then, under the dynamic event-triggered condition (12), N follower agents can track the leader agent and there is no Zeno behavior.

Proof. From equation (11) and condition (12), we have

$$\dot{\eta}_i(t) \ge -\beta_i \eta_i(t) - \frac{\xi_i}{\theta_i}\eta_i(t) \tag{13}$$

Thus

$$\eta_i(t) \ge \eta_i(0)e^{-\left(\beta_i + \frac{\xi_i}{\theta_i}\right)t} > 0 \tag{14}$$

Under the static event-triggered scheme, we consider the Lyapunov candidate:

$$V(t) = \sum_{i=1}^{N} z_i^T(t) P z_i(t) \tag{15}$$

Under the dynamic event-triggered scheme, we consider the Lyapunov candidate:

$$W(t) = V(t) + \sum_{i=1}^{n} \eta_i(t) \tag{16}$$

Then the derivative of $W(t)$ can be obtained as

$$\dot{W}(t) = \dot{V}(t) + \sum_{i=0}^{N} \dot{\eta}_t(t) \tag{17}$$

$\dot{V}(t)$ can be obtained as

$$\begin{aligned}\dot{V}(t) &= 2\sum_{i=1}^{N} z_i^T(t) P z_i(t) \\ &= \sum_{i=1}^{N} z_i^T(t)\left(PA + A^T P\right)z_i(t) \\ &\quad - 2\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij} z_i^T(t) PBK\left(\tilde{x}_i(t) - \tilde{x}_j(t)\right) \\ &\quad - 2\sum_{i=1}^{N} b_i z_i^T(t) PBK\left(\tilde{x}_i(t) - x_0(t)\right) \\ &= \sum_{i=1}^{N} z_i^T(t)\left(PA + A^T P\right)z_i(t) \\ &\quad - \sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\left(\tilde{x}_i(t) - \tilde{x}_j(t)\right)^T \Gamma\left(\tilde{x}_i(t) - \tilde{x}_j(t)\right) \\ &\quad + \sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\left(e_i(t) - e_j(t)\right)^T \Gamma\left(\tilde{x}_i(t) - \tilde{x}_j(t)\right) \\ &\quad - 2\sum_{i=1}^{N} b_i z_i^T(t)\Gamma z_i(t) - 2\sum_{i=1}^{N} b_i z_i^T(t)\Gamma e_i(t) \\ &= \sum_{i=1}^{N} z_i^T(t)\left(PA + A^T P\right)z_i(t) \\ &\quad - \sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\left(\tilde{x}_i(t) - \tilde{x}_j(t)\right)^T \Gamma\left(\tilde{x}_i(t) - \tilde{x}_j(t)\right) \\ &\quad + \sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\left(e_i(t) - e_j(t)\right)^T \Gamma\left(\tilde{x}_i(t) - \tilde{x}_j(t)\right)\end{aligned} \tag{18}$$

According to Lemma2, we have

$$\begin{aligned}-2\sum_{i=1}^{N} b_i z_i(t)^T \Gamma e_i(t) &\le \sum_{i=1}^{N} b_i z_i(t)^T \Gamma z_i(t) \\ &\quad + \sum_{i=1}^{N} b_i e_i(t)^T \Gamma e_i(t)\end{aligned} \tag{19}$$

and

$$\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\left(e_i(t)-e_j(t)\right)^T\Gamma\left(\tilde{x}_i(t)-\tilde{x}_j(t)\right) \le \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\left(e_i(t)-e_j(t)\right)^T\Gamma\left(e_i(t)-e_j(t)\right) + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\left(\tilde{x}_i(t)-\tilde{x}_j(t)\right)^T\Gamma\left(\tilde{x}_i(t)-\tilde{x}_j(t)\right) \quad (20)$$

Under the previous calculation, it follows that

$$\begin{aligned}\dot{V}(t)=&\sum_{i=1}^{N} z_i^T(t)\left(PA+A^TP\right)z_i(t)\\&-\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}\left(\tilde{x}_i(t)-\tilde{x}_j(t)\right)^T\Gamma\left(\tilde{x}_i(t)-\tilde{x}_j(t)\right)\\&+\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}\left(e_i(t)-e_j(t)\right)^T\Gamma\left(e_i(t)-e_j(t)\right)\\&-\sum_{i=1}^{N}b_i z_i^T(t)\Gamma z_i(t)-\sum_{i=1}^{N}b_i e_i^T(t)\Gamma e_i(t)\end{aligned} \quad (21)$$

Note that

$$\begin{aligned}&-\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}\left(\tilde{x}_i(t)-\tilde{x}_j(t)\right)^T\Gamma\left(\tilde{x}_i(t)-\tilde{x}_j(t)\right)\\&=-\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}\left(z_i(t)-z_j(t)\right)^T\Gamma\left(z_i(t)-z_j(t)\right)\\&=-\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}\left(e_i(t)-e_j(t)\right)^T\Gamma\left(e_i(t)-e_j(t)\right)\\&=-\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}\left(z_i(t)-z_j(t)\right)^T\Gamma\left(e_i(t)-e_j(t)\right)\end{aligned} \quad (22)$$

Using Young's inequality leads to

$$\begin{aligned}&-\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}\left(z_i(t)-z_j(t)\right)^T\Gamma\left(e_i(t)-e_j(t)\right)\\&\le\frac{1}{4}\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}\left(z_i(t)-z_j(t)\right)^T\Gamma\left(z_i(t)-z_j(t)\right)\\&+\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}\left(e_i(t)-e_j(t)\right)^T\Gamma\left(e_i(t)-e_j(t)\right)\end{aligned} \quad (23)$$

Substituting (23) into (22) yields

$$\begin{aligned}\dot{V}(t)\le&\sum_{i=1}^{N} z_i^T(t)\left(PA+A^TP\right)z_i(t)\\&-\frac{1}{4}\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}\left(z_i(t)-z_j(t)\right)^T\Gamma\left(z_i(t)-z_j(t)\right)\\&+\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}\left(e_i(t)-e_j(t)\right)^T\Gamma\left(e_i(t)-e_j(t)\right)\\&-\sum_{i=1}^{N}b_i z_i^T(t)\Gamma z_i(t)+\sum_{i=1}^{N}b_i e_i^T(t)\Gamma e_i(t)\end{aligned} \quad (24)$$

Then

$$\begin{aligned}\dot{V}(t)\le& z^T(t)\left[I_N\otimes\left(PA+A^TP\right)-\frac{1}{2}(L+D)\otimes\Gamma\right]z(t)\\&+\sum_{i=1}^{N}\left(\sum_{j=1}^{N}2a_{ij}+b_i\right)e_i^T(t)\Gamma e_i(t)\\&\le-\frac{\lambda_1}{8}\sum_{i=1}^{N}\left\|\sum_{j=1}^{N}a_{ij}\left(\tilde{x}_i(t)-\tilde{x}_j(t)\right)+b_i\left(\tilde{x}_i(t)-x_0(t)\right)\right\|^2\\&+\sum_{i=1}^{N}\left[\left(\sum_{j=1}^{N}2a_{ij}+b_i\right)\|\Gamma\|+\frac{\lambda_1}{4}\right]\|e_i(t)\|^2\end{aligned} \quad (25)$$

where λ_1 is the minimum eigenvalue of H.

If the triggering condition() is enforced for each agent, then

$$\dot{V}(t)\le-\sum_{i=1}^{N}(1-\sigma_i)\left\|\sum_{j=1}^{N}a_{ij}\left(\tilde{x}_i(t)-\tilde{x}_j(t)\right)+b_i\left(\tilde{x}_i(t)-x_0(t)\right)\right\|^2 \le 0 \quad (26)$$

Therefore, it implies that $z_i(t)\to 0,\ i=1,...,N$ as $t\to\infty$. That is, under the static event-triggered scheme, the tracking control problem of MAS (1), (2) is solved.

$$\begin{aligned}\dot{W}(t)&=\dot{V}(t)+\sum_{i=0}^{N}\dot{\eta}_t(t)\\&\le-\sum_{i=0}^{N}(1-\sigma_i)\left\|\tilde{z}_i(t)\right\|^2-\sum_{i=0}^{N}\beta_i\eta_i(t)-\sum_{i=0}^{N}\xi_i\frac{\eta_i(t)}{\theta_i}\\&=-\sum_{i=0}^{N}(1-\sigma_i)\left\|\tilde{z}_i(t)\right\|^2-(\beta_i+\frac{\xi_i}{\theta_i})\sum_{i=0}^{N}\eta_i(t)\\&=-k_W W(\mathrm{t})\end{aligned} \quad (27)$$

where $k_W=\min\left\{(1-\sigma_i)\left\|\tilde{z}_i(t)\right\|^2,\beta_i+\frac{\xi_i}{\theta_i}\right\}$, then

$$V(\mathrm{t})<\mathrm{W}(\mathrm{t})\le\mathrm{W}(0)\,\mathrm{e}^{-k_W t} \quad (28)$$

This implies that the MAS (1), (2) reaches consensus exponentially.

Next, we prove that there is no Zeno behavior.

Proof. Denote $\xi(t)=\frac{\|e(t)\|}{\|\tilde{z}(t)\|}$, in which $t\in\left[t_k^i,t_{k+1}^i\right)$, one has

$$\left\|\dot{\xi}(t)\right\|\le\frac{\|\dot{e}(t)\|}{\|\tilde{z}(t)\|}+\xi(t)\frac{\left\|\dot{\tilde{z}}(t)\right\|}{\|\tilde{z}(t)\|} \quad (29)$$

From (6), one gets

$$\dot{e}_i(t)=Ae_i(t)-BK\left[\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}\left(\tilde{z}_i(t)-\tilde{z}_j(t)\right)+b_i\tilde{z}_i(t)\right] \quad (30)$$

Then, we gets

$$\dot{\tilde{z}}(t)=(I_N\otimes A)\tilde{x}_i(t)-(I_N\otimes A)\tilde{x}_0(t)=(I_N\otimes A)\tilde{z}(t) \quad (31)$$

Then,

$$\|\dot{e}(t)\|\le\|I_N\otimes A\|\|e(t)\|+\|(L+D)\otimes BK\|\|\tilde{z}(t)\| \quad (32)$$

Consequently

$$\left\|\dot{\xi}(t)\right\|\le 2a\xi(t)+b \quad (33)$$

where $a=\|I_N\otimes A\|,\ b=\|(L+D)\otimes BK\|$.

Then $\xi(t)$ satisfies the bound

$$\xi(t)\le\phi(t,\phi_0) \quad (34)$$

where $\phi(t,\phi_0)$ is the solution of

$$\dot{\phi}(t,\phi_0)=2a\phi(t,\phi_0)+b \quad (35)$$

The corresponding solution of (35) during $t\in\left[t_k^i,t_{k+1}^i\right)$ is:

$$\phi(t,\phi_0)=\frac{b}{2a}e^{2a\tau}-\frac{b}{2a} \tag{36}$$

Hence the inter-event interval are bounded from below by the time τ that satisfies

$$\phi(\tau,0)=\frac{b}{2a}e^{2a\tau}-\frac{b}{2a} \tag{37}$$

From the equation (12), the next inter-event interval of agent i is bounded from below by a time τ that satisfies

$$\frac{b}{2a}e^{2a\tau}-\frac{b}{2a}=\sqrt{\frac{\frac{\eta_i(t)}{\theta_i}+\sigma_i\lambda_1}{4\gamma\|\Gamma\|+2\lambda_1}} \tag{38}$$

Then the inter-event interval τ is

$$\tau=\frac{1}{2a}In\left(1+\frac{2ad}{b}\right) \tag{39}$$

where $d=\sqrt{\frac{\frac{\eta_i(t)}{\theta_i}+\sigma_i\lambda_1}{4\gamma\|\Gamma\|+2\lambda_1}}$.

Because $a,b,d>0$,then $\tau>0$. Thus the inter-event interval for agent i is strictly greater than zero based on (29). In summary, we can conclude that the Zeno behavior is not exhibited by the above analysis.

4. SIMULATION

In this section, the developed dynamic event-triggered scheme design approach is applied to address a leader-following control problem for a group of five autonomous mobile robots. The robot 0 is the leader and the four robots 1-4 are followers. Denote $x_i(t)=col\{\bar{x}_i(t),\bar{v}_i(t)\}$, where $\bar{x}_i(t)\in\mathbb{R}$ and $\bar{v}_i(t)$ are the coordinates of the center of mass of the i-th robot and the linear velocity components along the X axes, respectively.

The initial states of five agents are described by $S_0=col\{1,0\}, S_1=col\{2,-1\}, S_2=col\{-1,1\}, S_3=col\{2,1\}$, $S_4=col\{3,1\}$. The desired states of the four followers are described by $F_1=col\{1,0\}, F_2=col\{1,0\}, F_3=col\{1,0\}$, $F_4=col\{1,0\}$.

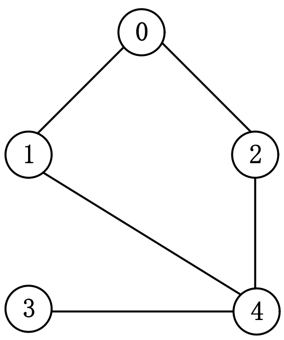

Fig. 1 Communication topology of general linear system

$A=[0\ 1;0\ 0]$, $B=[0,1]$. The communication topology is described in Fig. 1. It is easy to check that the graph is connected.

$$H=\begin{bmatrix}2&0&-1&0\\0&2&-1&0\\-1&-1&3&-1\\0&0&-1&1\end{bmatrix}$$

The eigenvalues of H are 0.3249, 1.4608, 2.0000 and 4.2143 respectively. Obviously, λ_1=0.3249 .

By solving the LMI with $Q=I_2$ gives P=[0.7474, 0.5402; 0.5402, 1.0570], then the feedback gain matrices K and Γ are obtained as K=[0.5402, 1.0570] and Γ =[0.2918, 0.5710; 0.5710, 1.1173]. The constant scalars are selected as $\xi_i=1$, $\theta_i=0.5$, $\sigma_i=0.35$, $\beta=0.1$. The simulation time is 40s and the step size is 0.01s. Suppose that events are triggered for all agents at the beginning of the simulation. Fig. 2 shows the state evolution under the static event-triggered scheme (10). Fig. 3 shows the state evolution under the dynamic event-triggered scheme (12). It can be seen under the dynamic event-triggered scheme, the consensus of MAS(1), (2) can be achieved. And the agents achieve consensus at 30 seconds under the dynamic event-triggered scheme (12). But the agents have not achieve consensus at 30 seconds under the static event-triggered scheme (10). It indicates that the convergence rate of using the dynamic event-triggered scheme (12) is faster than using the static event-triggered scheme (10). Fig. 4 shows the triggering errors under the dynamic event-triggered law (12). We can see that the threshold goes to zero with an overall decreasing tendency. Table. 1 shows the triggering numbers under the static triggering scheme (10) and the dynamic triggering scheme (12). It can be seen under the static triggering scheme, the triggering number of an agent will reduce significantly.

From the simulation results we can know that the proposed dynamic event-triggered scheme can fulfill the consensus task for MAS (1), (2) and has a satisfactory control performance with reducing both communication and computation resource.

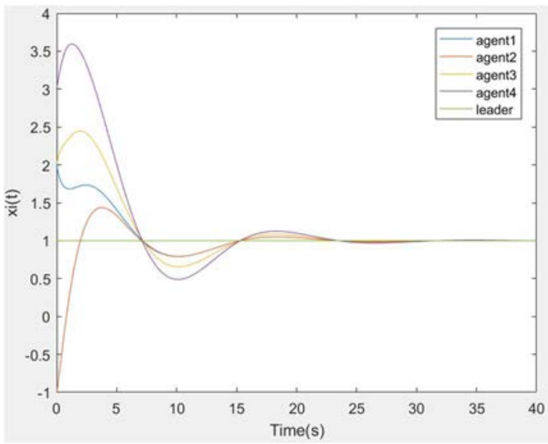

Fig. 2 The state evolution under the static event-triggered scheme (10)

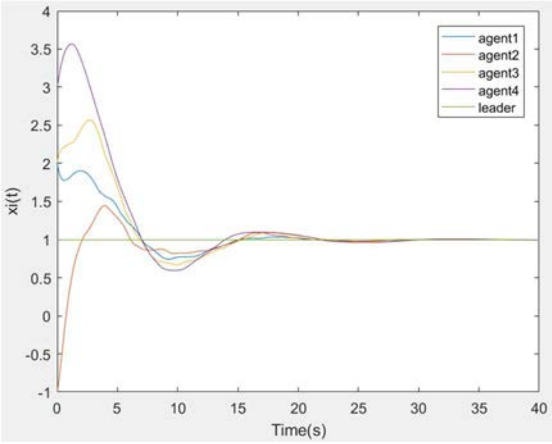

Fig. 3 The state evolution under the dynamic event-triggered scheme (12)

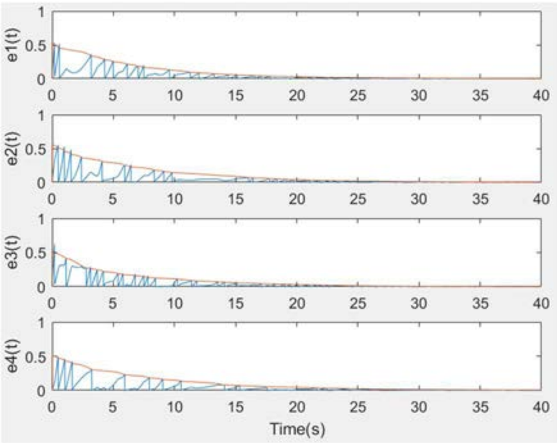

Fig. 4 The triggering errors under the dynamic triggering scheme

Table. 1 Triggering times under the static triggering scheme (10) and the dynamic triggering scheme (12)

Title	Agent 1	Agent 2	Agent 3	Agent 4
Triggering numbers under the static triggering law (10)	291	284	252	260
Triggering numbers under the dynamic triggeringlaw (12)	31	36	48	20

5. CONCLUSION

For saving both communication and computation energy, a dynamic event-triggered scheme is proposed for leader-following general linear systems in this paper. We show that if the communication graph is undirected and contains a directed spanning tree, leader-following consensus is achieved and there is no Zeno behavior. Numerical simulations illustrate the effectiveness of the theoretical results and the results show that the proposed dynamic event-triggered scheme can fulfill the consensus task for MAS (1), (2) and has a satisfactory control performance with reducing the triggering numbers for each agent. It means the proposed dynamic triggering scheme lead to reduction of actuation updates、inter-agent communications and computation resource. In the future, the consensus problem of multi-agent systems with disturbances and delays will be further studied.

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