Paper:

Distributed finite-time practical consensus of second-order multi-agent system by event-triggered strategy

Wei Han[1], Pingli Lu[1]*, Shaopan Zhang[1], Haikuo Liu[1], Changkun Du[1], Chao Li[2]

1. School of Automation, Beijing Institute of Technology, Beijing, PR China
2. School of Electrical and Control Engineering, North China University of technology, Beijing, PR China
*Corresponding Author: pinglilu@bit.edu.cn

Abstract. This paper considers the finite-time and distributed event-triggered consensus control for multi-agent system (MAS) with second-order dynamics under a undirected graph. A novel event-triggered function, which depends only on local information, is adopted. By employing a novel distributed event-triggered controller for each agent, practical finite-time consensus (FTC) of MAS can be achieved without continuous communication. A sufficient condition is obtained through the analysis by Lyapunov theory and graph theory. In addition, the feasibility of the proposed approach is guaranteed by comprehensive theoretical demonstration of practical FTC and analysis of the Zeno behavior. Finally, a numerical simulation is provided to illustrate the correctness of theoretical results.

Keywords: Second-order multi-agent system, Finite-time, Distributed event-triggered, Zeno behavior

1. Introduction

In recent years, various issues of multi-agent system including consensus, formation and tracking, have attracted a lot of attention due to their wide application, such as UAVs cooperative control[1, 2], formation control[3, 4], social insects[5, 6], group computing[7, 8], etc. As a basic problem of consensus, the distributed consensus problem is that how to design a distributed controller and let all agents of the network achieve consensus through the local information exchange.

With the in-depth study of the consensus problem, we realize that the convergence rate is the most important performance factor reflecting the design quality of the protocol. Many researchers study and improve the convergence rate by increasing the coupling strength, optimizing the communication weight and designing the optimal network topology[9–14]. However, these protocols[5, 12, 15–17] can only make the system asymptotically convergent. Only when the time tends to infinity, the algorithm based on asymptotic consensus is expected to achieve the goal. In many practical systems, especially the mechanical systems, since the high control accuracy is required, it is of great significance to achieve finite-time consensus. Compared with the asymptotic convergence, the finite-time consensus has advantageous dynamic properties, for instance, higher accuracy, faster convergence rate and good robustness against nonlinear uncertainties. Due to the above advantages, the finite time consensus protocol is studied in [4, 18–23]. Bhat[19] provides a strict basis for the FTC theory of continuous systems and gives an estimate of the stabilization time.

Note that the above literature only focuses on convergence time, which may lead to frequent information exchange, resulting in unnecessary energy consumption and reduced communication efficiency. Actually, the frequent information exchange among agents may be infeasible in many real-world applications because of the inadequate communication resources of the communication devices equipped on agents. Therefore, the mechanism of event-triggered control has also been widely studied[24–32]. Zhang[33] studied the FTC problem of first-order multi-agent system and proposed a distributed event-triggered protocol. Wang, Li and Xing[34] proposed an finite-time event-triggered control protocol to solve average consensus problem and the relationship between the initial state and the convergence time. [35, 36] investigated event-triggered FTC algorithm which can adjust the desired convergence time for first-order multi-agent systems. Moreover, for second-order multi-agent systems, Lu et al. investigated FTC of MAS with event-triggered function[37]. Qian[38] proposed a distributed event-triggered protocol for practical FTC problem of second-order multi-agent system, and excluded Zeno behavior. However, undesirable continuous information exchange is still needed to check the triggering conditions in the aforementioned design methods. It is desirable that the need for continuous information exchange with neighbors in its own event detection should be avoided.

Inspired by the facts and challenges stated above, it is of particular interest to study the practical FTC of the second-order multi-agent system without continuous information exchange. A finite-time control protocol is proposed under the distributed event-triggered strategy in this paper. Moreover, under the proposed strategy, the continuous communication is not required among agents and the Zeno behavior is excluded. The primary contributions of the paper are summarized as follows.

- Compared with the traditional asymptotic consensus control, the second-order MAS can achieve consensus in a finite time under the protocol of this paper, improve the control accuracy of the system, and more easily meet the practical applications.

- Compared with Qian[38], in this paper, the communication among the agents is fully intermittent, that is, not only in the control protocol, but also in the event-triggered function, the agent does not need continuous information exchange with its neighbors to achieve FTC.

The rest of the paper is arranged as follows. In Section 2, we give some useful preparatory knowledge and lemmas, and give the FTC problem to be solved. A new practical FTC protocol under distributed event-triggered strategy of second-order multi-agent system is proposed in Section 3. Numerical examples and conclusions are given in Section 4 and 5 respectively.

2. Preliminaries and Problem Formulation

In this paper, let $\mathbf{1}_n = [1,\dots,1]^T$. Let $\mathrm{sig}(x)^\alpha = \mathrm{sign}(x)\,|x|^\alpha$, where $\alpha > 0$, $x \in R$ and $\mathrm{sign}(\cdot)$ is the standard signum function. For a vector $x = [x_1,\dots,x_n]^T$, denote $\mathrm{sig}(x)^\alpha = [\mathrm{sig}(x_1)^\alpha,\dots,\mathrm{sig}(x_n)^\alpha]^T$ and $|x| = [|x_1|,\dots,|x_n|]^T$.

2.1. Algebraic Graph Theory

For the multi-agent system, assume that each agent is a node and the information exchange of n agents is denoted by an undirected graph $\mathscr{G} = (\mathscr{V},\mathscr{E},\mathscr{A})$, where $\mathscr{V} = \{1,2,\dots,n\}$ is a set of nodes , $\mathscr{E} = \mathscr{V}\times\mathscr{V}$ is a set of edges and $\mathscr{A} = [a_{ij}] \in R^{n\times n}$ is a weighted adjacency matrix with weights $a_{ij} \geq 0$ for $\forall i,j \in \mathscr{V}$. If there is an edge between agent i and j, i.e.,$(i,j)\in\mathscr{E}$, then $a_{ij} = a_{ji} > 0$. Moreover, $a_{ii} = 0$ for all $i \in \mathscr{V}$ means that there are no self-loops. If there exists an edge between agent i and agent j, then agent j is called a neighbor of agent i. The set of neighbors of agent i is denoted by $\mathscr{N}_i = \{j : (i,j)\in\mathscr{E}\}$. The Laplacian matrix $L(A)$ of graph $\mathscr{G}$ is denoted as $l_{ij} = -a_{ij}$ for $i \neq j$ and $l_{ii} = \Sigma_{j=1,j\neq i}^n a_{ij}$. Obviously, $\Sigma_{j=1}^n l_{ij} = 0$ for all $i \in \mathscr{V}$. A directed path in the graph $\mathscr{G}$ from agent i to agent j is a sequence of distinct vertices starting with agent i and ending with agent j such that agent j is reachable from agent i. The graph $\mathscr{G}$ is connected if there is a path between any two agents.

2.2. Some Lemmas

Lemma 1. [39] Suppose there exists a positive definite continuous function $V(t): R^n \to R$, real numbers $0 < \eta < 1$, $c > 0$ and $d \geq 0$, and an open neighborhood $R_0^n \subset R^n$ of the origin such that

$$\dot{V}(t) \leq -cV(t)^\eta + d,$$

then $V(t)$ is finite-time bounded. In addition, the settling time T, depending on the initial state $V(0)$, is determined as

$$T = \frac{V(0)^{1-\eta}}{c(1-\eta)}.$$

Lemma 2. [22] For $x \in R$, and $\alpha \in R$, then

$$\frac{d\,|x|^{1+\alpha}}{dt} = (1+\alpha)\mathrm{sig}(x)^\alpha\frac{dx}{dt}.$$

Lemma 3. [33] For $x_i \in R$, $i = 1,\dots,n$, $0 < p \leq 1$, then

$$(\sum_{i=1}^n |x_i|)^p \leq \sum_{i=1}^n |x_i|^p .$$

Lemma 4. [33] For $M = \frac{1}{2}(L+L^T)$, Γ is a bounded closed set satisfying $\Gamma = \{\delta \in R : \delta^T\delta = 1$ and $\delta = \beta\,|\omega|^\alpha$ for some $\omega \perp \mathbf{1}_n\}$, and the function $\delta^T M\delta$ is continuous with regard to δ and for any $\delta \in \Gamma$, we have $\delta^T M\delta \neq 0$, then there exist $\min_{\delta\in\Omega}\delta^T M\delta$, denoted by ε which is larger than 0.

2.3. Problem Formulation

For a second-order multi-agent system, which consists of n continuous-time dynamic agents and all the agents share a common state space R, the dynamics of the i-th agent is described as

$$\dot{x}_i = v_i, \quad \dot{v}_i = u_i, \quad i = 1,\dots,n, \tag{1}$$

where x_i is the position state of the i-th agent, v_i is the velocity state of the i-th agent, u_i is a control protocol which needs to be designed.

Assumption 1. The communication graph of MAS (1) is undirected and connected.

Remark 1. For Assumption 1, the Laplacian has a single zero eigenvalue and the corresponding eigenvector is $\mathbf{1}_n$, $\mathbf{1}_n^T L = 0$.

Definition 1. Practical FTC: There exists $\varepsilon_1 > 0, \varepsilon_2 > 0$ and a finite-time T which satisfy

$$\left|x_i(t) - x_j(t)\right| < \varepsilon_1, \left|v_i(t) - v_j(t)\right| < \varepsilon_2,$$

for $t \geq T$ and $i,j \in V$, it is said that the MAS (1) reach practical FTC.

3. Main Results

In this subsection, in order to save the communication resources, our control goal is to let all agents of MAS (1) satisfying Assumption 1 reach practical FTC. In order to introduce the decentralized event-triggered strategy for MAS (1), it is assumed that the triggering time sequence at which agent i measures its states and obtains its neighbors' states is denoted by $\{t_0^i, t_1^i, \dots, t_k^i, \dots\}$, and each agent can only exchange the information of states with its neighbors. For agent i with dynamics(1), state estimates are defined as follows.

$$\begin{cases} \tilde{x}_i(t) = x_i(t_k^i) + v_i(t_k^i)(t - t_k^i) \\ \tilde{v}_i(t) = v_i(t_k^i) \end{cases} \tag{2}$$

A practical FTC protocol of MAS (1) with distributed event-triggered strategy of agent i is designed as follows.

$$\begin{aligned} u_i(t) = &-\beta \mathrm{sig}(\sum_{j\in\mathcal{N}_i} a_{ij}[(\tilde{x}_i(t)-\tilde{x}_j(t)) \\ &+\gamma(\tilde{v}_i(t)-\tilde{v}_j(t))])^{\alpha} - \frac{1}{\gamma}v_i(t), i\in V, \end{aligned} \quad (3)$$

for $t\in[t_k^i, t_{k+1}^i), k=0,1,\dots,$ where $0<\alpha<1, \beta>0$ and $\gamma>0$.

Design the distributed event-triggered function for agent i as:

$$\begin{aligned} \beta|\sum_{j\in\mathcal{N}_i} a_{ij}[(\tilde{x}_i(t)-\tilde{x}_j(t))+\gamma(\tilde{v}_i(t)-\tilde{v}_j(t))]|^{\alpha} \\ < \lambda e^{-\kappa t}+\delta \end{aligned} \quad (4)$$

where $\kappa>0$, $\lambda>0$ and $0<\delta<\dfrac{2\varepsilon}{\beta}$. ε will be introduced later.

Then MAS (1) can be rewritten as follows.

$$\begin{cases} \dot{x}_i(t) = v_i(t) \\ \dot{v}_i(t) = -\beta \mathrm{sig}(\sum_{j\in\mathcal{N}_i} a_{ij}[(\tilde{x}_i(t)-\tilde{x}_j(t)) \\ \qquad +\gamma(\tilde{v}_i(t)-\tilde{v}_j(t))])^{\alpha} - \frac{1}{\gamma}v_i(t) \end{cases}, i\in V.$$

The distributed control protocol (2) can be transformed into

$$\begin{aligned} u_i(t) = &-\beta \mathrm{sig}(\sum_{j\in\mathcal{N}_i} a_{ij}[(\tilde{x}_i(t)-\tilde{x}_j(t)) \\ &+\gamma(\tilde{v}_i(t)-\tilde{v}_j(t))])^{\alpha} - \frac{1}{\gamma}v_i(t) \\ = &-\beta \mathrm{sig}(\sum_{j\in\mathcal{N}_i} a_{ij}[(x_i(t)-x_j(t))+\gamma(v_i(t) \\ &-v_j(t))+(x_i(t_k^i)-x_i(t))-(x_j(t_k^j) \\ &-x_j(t))+\gamma(v_i(t_k^i)-v_i(t))-(v_j(t_k^j) \\ &-v_j(t))+v_i(t_k^i)(t-t_k^i)-v_j(t_k^j)(t-t_k^j)])^{\alpha} \\ &-\frac{1}{\gamma}v_i(t). \end{aligned}$$

Let

$$\begin{aligned} y_{x_i}(t) &= \sum_{j\in\mathcal{N}_i} a_{ij}(x_i(t)-x_j(t)), \\ y_{v_i}(t) &= \sum_{j\in\mathcal{N}_i} a_{ij}(v_i(t)-v_j(t)), \\ E_{x_i}(t) &= \sum_{j\in\mathcal{N}_i} a_{ij}[(x_i(t_k^i)-x_i(t))-(x_j(t_k^j)-x_j(t))], \end{aligned}$$

$$E_{v_i}(t) = \sum_{j\in\mathcal{N}_i} a_{ij}[(v_i(t_k^i)-v_i(t))-(v_j(t_k^j)-v_j(t))],$$

then

$$\begin{aligned} u_i(t) = &-\beta \mathrm{sig}(y_{x_i}(t)+\gamma y_{v_i}(t)+E_{x_i}(t)+\gamma E_{v_i}(t) \\ &+\sum_{j\in\mathcal{N}_i} a_{ij}(v_i(t_k^i)(t-t_k^i)-v_j(t_k^j)(t-t_k^j)))^{\alpha} \\ &-\frac{1}{\gamma}v_i(t) \\ = &-\beta \mathrm{sig}(y_{x_i}(t)+\gamma y_{v_i}(t)+E_{x_i}(t)+\gamma E_{v_i}(t) \\ &+g_i(t))-\frac{1}{\gamma}v_i(t), \end{aligned}$$

where $g_i(t)=\sum_{j\in\mathcal{N}_i} a_{ij}(v_i(t_k^i)(t-t_k^i)-v_j(t_k^j)(t-t_k^j))$.

Theorem 1. Consider the second-order MAS (1) with undirected and connected graph, the position errors and the velocity errors between each agent will reach practical FTC under distributed control protocol (3) and event-triggered function (4).

Proof. For MAS (1), construct the following Lyapunov function:

$$V(t)=\sum_{i=1}^{n}\frac{\beta}{1+\alpha}|y_{x_i}(t)+\gamma y_{v_i}(t)|^{(1+\alpha)}. \quad (5)$$

Consider the derivative of V(t),

$$\begin{aligned} \dot{V}(t) = &\sum_{i=1}^{n}\beta \mathrm{sig}(y_{x_i}(t)+\gamma y_{v_i}(t))^{\alpha}(\dot{y}_{x_i}(t)+\gamma\dot{y}_{v_i}(t)) \\ = &\sum_{i=1}^{n}\beta \mathrm{sig}(y_{x_i}(t)+\gamma y_{v_i}(t))^{\alpha}(y_{v_i}(t)+\gamma L_i\dot{v}(t)) \\ = &\sum_{i=1}^{n}\beta \mathrm{sig}(y_{x_i}(t)+\gamma y_{v_i}(t))^{\alpha}(L_i v(t) \\ &+\gamma L_i(-\beta \mathrm{sig}(y_x(t)+\gamma y_v(t)+E_x(t) \\ &+\gamma E_v(t)+G(t))^{\alpha})-\frac{1}{\gamma}v_i(t)) \\ = &-\gamma\sum_{i=1}^{n}\beta \mathrm{sig}(y_{x_i}(t)+\gamma y_{v_i}(t))^{\alpha}L_i\beta \mathrm{sig}(y_x(t) \\ &+\gamma y_v(t)+E_x(t)+\gamma E_v(t)+G(t))^{\alpha} \\ = &-\gamma\sum_{i=1}^{n}\sum_{j\in\mathcal{N}_i}\beta \mathrm{sig}(y_{x_i}(t) \\ &+\gamma y_{v_i}(t))^{\alpha} l_{ij}\beta \mathrm{sig}(y_{x_j}(t)+\gamma y_{v_j}(t)+E_{x_j}(t) \\ &+\gamma E_{v_j}(t)+g_j(t))^{\alpha}, \end{aligned}$$

where

$$\begin{aligned} &L_i=[l_{i1},l_{i2},\dots,l_{in}], G(t)=[g_1(t),g_2(t),\dots,g_n(t)]^T, \\ &y_x=[y_{x_1},y_{x_2},\dots,y_{x_n}]^T, y_v=[y_{v_1},y_{v_2},\dots,y_{v_n}]^T, \\ &E_x=[E_{x_1},E_{x_2},\dots,E_{x_n}]^T, E_v=[E_{v_1},E_{v_2},\dots,E_{v_n}]^T. \end{aligned}$$

Because of

$$\mathrm{sig}(y_{x_i}(t)+\gamma y_{v_i}(t)) \le |y_{x_i}(t)+\gamma y_{v_i}(t)|,$$

then we have

$$\begin{aligned}&\mathrm{sig}(y_{x_j}(t)+\gamma y_{v_j}(t)+E_{x_j}(t)+\gamma E_{v_j}(t)+g_j(t))^\alpha\\ \le&\left|y_{x_j}(t)+\gamma y_{v_j}(t)+E_{x_j}(t)+\gamma E_{v_j}(t)+g_j(t)\right|^\alpha\\ \le&\left|y_{x_j}(t)+\gamma y_{v_j}(t)\right|^\alpha+\left|E_{x_j}(t)+\gamma E_{v_j}(t)+g_j(t)\right|^\alpha.\end{aligned}$$

For $i\neq j, j\in\mathscr{N}_i, l_{ij}<0$, which means

$$\begin{aligned}&\sum_{i=1}^{n}\sum_{j\in\mathscr{N}_i}\beta\,\mathrm{sig}(y_{x_i}(t)+\gamma y_{v_i}(t))^\alpha l_{ij}\beta\,\mathrm{sig}(y_{x_j}(t)\\ &+\gamma y_{v_j}(t)+E_{x_j}(t)+\gamma E_{v_j}(t)+g_j(t))^\alpha\\ \ge&\sum_{i=1}^{n}\sum_{j\in\mathscr{N}_i}\beta\left|y_{x_i}(t)+\gamma y_{v_i}(t)\right|^\alpha l_{ij}\beta(\left|y_{x_j}(t)\right.\\ &\left.+\gamma y_{v_j}(t)\right|^\alpha+\left|E_{x_j}(t)+\gamma E_{v_j}(t)+g_j(t)\right|^\alpha),\end{aligned}\tag{6}$$

$$\begin{aligned}&\left|E_{x_i}(t)+\gamma E_{v_i}(t)+g_i(t)\right|^\alpha\\ =&|\sum_{j\in\mathscr{N}_i}a_{ij}[(\tilde{x}_i(t)-\tilde{x}_j(t))+\gamma(\tilde{v}_i(t)-\tilde{v}_j(t))\\ &-(x_i(t)-x_j(t))-\gamma(v_i(t)-v_j(t))]|^\alpha\\ \le&|\sum_{j\in\mathscr{N}_i}a_{ij}[(\tilde{x}_i(t)-\tilde{x}_j(t))+\gamma(\tilde{v}_i(t)-\tilde{v}_j(t))]|^\alpha\\ &+|y_{x_i}+\gamma y_{v_i}|^\alpha.\end{aligned}\tag{7}$$

According to (4), (6) and (7), we can obtain

$$\begin{aligned}\dot{V}(t)\le&-\gamma\sum_{i=1}^{n}\sum_{j\in\mathscr{N}_i}\beta\left|y_{x_i}(t)+\gamma y_{v_i}(t)\right|^\alpha l_{ij}\beta(2\left|y_{x_j}(t)\right.\\ &\left.+\gamma y_{v_j}(t)\right|^\alpha+|\sum_{j\in\mathscr{N}_i}a_{ij}[(\tilde{x}_i(t)-\tilde{x}_j(t))\\ &+\gamma(\tilde{v}_i(t)-\tilde{v}_j(t))]|^\alpha)\\ \le&-\gamma\sum_{i=1}^{n}\sum_{j\in\mathscr{N}_i}\beta\left|y_{x_i}(t)+\gamma y_{v_i}(t)\right|^\alpha l_{ij}\beta(2\left|y_{x_j}(t)\right.\\ &\left.+\gamma y_{v_j}(t)\right|^\alpha+\lambda e^{-\kappa t}+\delta)\\ \le&-2\gamma\sum_{i=1}^{n}\sum_{j\in\mathscr{N}_i}\beta\left|y_{x_i}(t)+\gamma y_{v_i}(t)\right|^\alpha l_{ij}\beta\left|y_{x_j}(t)\right.\\ &\left.+\gamma y_{v_j}(t)\right|^\alpha-\gamma\beta^2(\lambda e^{-\kappa t}+\delta)\sum_{i=1}^{n}\sum_{j\in\mathscr{N}_i}l_{ij}\left|y_{x_i}(t)\right.\\ &\left.+\gamma y_{v_i}(t)\right|^\alpha\\ \le&-2\gamma(\beta\left|y_x(t)+\gamma y_v(t)\right|^\alpha)^T L(\beta\left|y_x(t)\right.\\ &\left.+\gamma y_v(t)\right|^\alpha)-\gamma\beta^2(\lambda e^{-\kappa t}+\delta)\sum_{i=1}^{n}\sum_{j\in\mathscr{N}_i}l_{ij}\left|y_{x_i}(t)\right.\\ &\left.+\gamma y_{v_i}(t)\right|^\alpha.\end{aligned}$$

According to lemma 4, for $y_x(t)+\gamma y_v(t)\perp\mathbf{1}_n$, then

$$\begin{aligned}&(\beta\left|y_x(t)+\gamma y_v(t)\right|^\alpha)^T L(\beta\left|y_x(t)+\gamma y_v(t)\right|^\alpha)\\ &\ge\min_{\xi\in\Omega}\xi^T M\xi\triangleq\varepsilon>0.\end{aligned}\tag{8}$$

Let $b_i=-\sum_{j\in\mathscr{N}_i}l_{ij}$, $b_{\max}=\max_{i\in V}b_i$, then

$$\begin{aligned}\dot{V}(t)\le&-2\gamma(\beta\left|y_x(t)+\gamma y_v(t)\right|^\alpha)^T L(\beta\left|y_x(t)\right.\\ &\left.+\gamma y_v(t)\right|^\alpha)+\gamma\beta^2(\lambda e^{-\kappa t}+\delta)\sum_{i=1}^{n}b_i\left|y_{x_i}(t)\right.\\ &\left.+\gamma y_{v_i}(t)\right|^\alpha\\ \le&-2\gamma\varepsilon\sum_{i=1}^{n}\beta^2\left|y_{x_i}(t)+\gamma y_{v_i}(t)\right|^{2\alpha}+\gamma\beta(\lambda e^{-\kappa t}\\ &+\delta)\sum_{i=1}^{n}(\frac{\beta^2\left|y_{x_i}(t)+\gamma y_{v_i}(t)\right|^{2\alpha}}{2}+\frac{b_i^2}{2})\\ \le&-2\gamma\varepsilon\sum_{i=1}^{n}\beta^2\left|y_{x_i}(t)+\gamma y_{v_i}(t)\right|^{2\alpha}+\gamma\beta(\lambda e^{-\kappa t}\\ &+\delta)\sum_{i=1}^{n}\frac{\beta^2\left|y_{x_i}(t)+\gamma y_{v_i}(t)\right|^{2\alpha}}{2}\\ &+(\lambda e^{-\kappa t}+\delta)\frac{n\gamma\beta b_{\max}^2}{2}\\ =&-\gamma[2\varepsilon-\beta(\lambda e^{-\kappa t}+\delta)]\sum_{i=1}^{n}\beta^2\left|y_{x_i}(t)+\gamma y_{v_i}(t)\right|^{2\alpha}\\ &+(\lambda e^{-\kappa t}+\delta)\frac{n\gamma\beta b_{\max}^2}{2}\\ =&-\gamma[2\varepsilon-\beta(\lambda e^{-\kappa t}+\delta)](1+\alpha)^{\frac{2\alpha}{1+\alpha}}\beta^{\frac{2}{1+\alpha}}\\ &\times(\sum_{i=1}^{n}\frac{\beta}{1+\alpha}\beta^2\left|y_{x_i}(t)+\gamma y_{v_i}(t)\right|^{1+\alpha})^{\frac{2\alpha}{1+\alpha}}\\ &+(\lambda e^{-\kappa t}+\delta)\frac{n\gamma\beta b_{\max}^2}{2}.\end{aligned}$$

Because $\lambda e^{-\kappa t}$ is bounded about t, then we can obtain

$$\begin{aligned}\dot{V}(t)\le&-\gamma(2\varepsilon-\beta\delta)(1+\alpha)^{\frac{2\alpha}{1+\alpha}}\beta^{\frac{2}{1+\alpha}}\\ &\times(\sum_{i=1}^{n}\frac{\beta}{1+\alpha}\beta^2\left|y_{x_i}(t)+\gamma y_{v_i}(t)\right|^{1+\alpha})^{\frac{2\alpha}{1+\alpha}}\\ &+\frac{n\gamma\beta\delta b_{\max}^2}{2}.\end{aligned}$$

Let $C=\gamma(2\varepsilon-\beta\delta)(1+\alpha)^{\frac{2\alpha}{1+\alpha}}\beta^{\frac{2}{1+\alpha}}$, $D=\dfrac{n\gamma\beta\delta b_{\max}^2}{2}$ and $\mu=\dfrac{2\alpha}{1+\alpha}$, we get $D>0$ and $0<\mu<1$. Set δ such that $0<\delta<\dfrac{2\varepsilon}{\beta}$ and $C>0$, then we have

$$\dot{V}(t)\le-CV(t)^\mu+D.\tag{9}$$

According to Lemma 1, $V(t)$ is finite-time bounded. Thus MAS (1) will reach practical FTC in finite time $T=\frac{V(0)^{1-\mu}}{C(1-\mu)}$.

Remark 2. According to Theorem 1, the problem of practical FTC with distributed event-triggered strategy for MAS (1) can be solved. Compared with [38], the proposed distributed event-triggered strategy can work without continuous information exchange.

Let $\Theta(t) = \beta|\sum_{j\in\mathcal{N}_i} a_{ij}[(\tilde{x}_i(t) - \tilde{x}_j(t)) + \gamma(\tilde{v}_i(t) - \tilde{v}_j(t))]|^{\alpha}$, take the derivative of $\Theta(t)$ as follow.

$$\begin{aligned}\dot{\Theta}(t) =&\beta\mathrm{sig}(\sum_{j\in\mathcal{N}_i} a_{ij}[(\tilde{x}_i(t)-\tilde{x}_j(t))+\gamma(\tilde{v}_i(t)\\ &-\tilde{v}_j(t))])^{\alpha-1}(v_i(t_k^i)-v_j(t_k^j))\\ \leq&\beta|\sum_{j\in\mathcal{N}_i} a_{ij}[(\tilde{x}_i(t)-\tilde{x}_j(t))+\gamma(\tilde{v}_i(t)\\ &-\tilde{v}_j(t))]|^{\alpha-1}(v_i(t_k^i)-v_j(t_k^j))\\ \leq&(\lambda e^{-\kappa t}+\delta)^{\frac{\alpha-1}{\alpha}}(v_i(t_k^i)-v_j(t_k^j)).\end{aligned}$$

By the above analysis, the MAS (1) can reach practical FTC, that means the position states and the velocity states are bounded. $\dot{\Theta}(t)$ is related to states $v_i(t_k^i)$, $v_j(t_k^j)$ and the parameters $\alpha, \kappa, \lambda, \delta$. Therefore, we can know that the growth of $\Theta(t)$ is bounded. Since $\lim_{t\to\infty}(\lambda e^{-\kappa t}+\delta) = \delta$, there exists a lower bound of time interval $\tau = t_{k+1} - t_k$ between two consecutive event-triggered times

$$\beta|\sum_{j\in\mathcal{N}_i} a_{ij}[(\tilde{x}_i(t)-\tilde{x}_j(t))+\gamma(\tilde{v}_i(t)-\tilde{v}_j(t))]|^{\alpha} < \delta.(10)$$

Then the Zeno behavior can be excluded.

Remark 3. Obviously, the constant δ is associated with the consensus area. That is to say, if $\delta > 0$, MAS (1) can only reach the practical FTC. If δ gets smaller, then the consensus area can be smaller. Specially, if $\delta = 0$, MAS (1) can reach the FTC, but it is impossible to exclude the Zeno behavior.

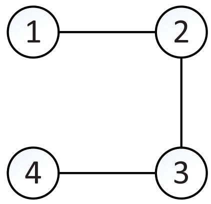

Fig. 1. Communication topology of second-order multi-agent system

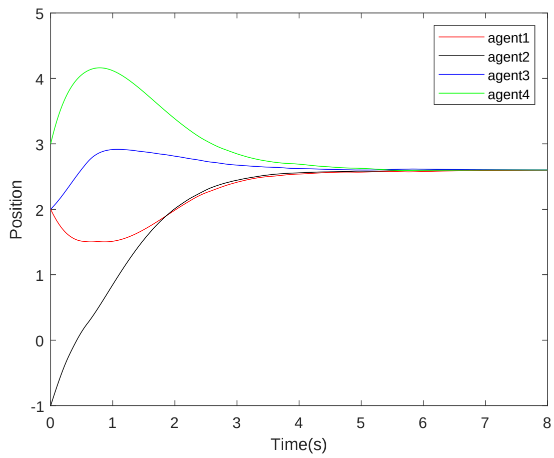

Fig. 2. The trajectories of all agents' position

4. Numerical Example

In this section, we give an example to illustrate the validity of the FTC protocol based on distributed event-based strategy. For 2D or 3D cases, we can decompose it into a combination of single dimension consensus problem. Therefore, we consider FTC problem with event-triggered in one-dimensional case. Assume that there are four agents of the second-order multi-agent system, the dynamics of each agent satisfies MAS (1). Consider the case of network topology in Fig.1 and the corresponding adjacent matrix is as follows.

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

We choose $\alpha = 0.5$, $\beta = 2.2$, $\gamma = 0.69$, $\lambda = 100$, $\kappa = 0.02$, $\delta = 0.01$ and the initial states of each agent are $x(0) = [2,-1,2,3]^T$, $v(0) = [-2,3,1,4]^T$.

The state trajectories are presented in Figs.2-3. It can be seen that each agent of the second-order multi-agent system reaches a small bounded consensus area in finite time, which is associated with parameter δ. Figs.4-5 show the change of position error and velocity error of each agent, respectively. Fig.6 show the events of all agents and the interval between two consecutive event-triggered times has a lower limit. Furthermore, the Zeno behavior is excluded. By simulation results, the proposed approach can fulfill practical FTC under distribute event-triggered strategy of the second-order multi-agent system, and have satisfactory control performance with saving communication resources.

5. Conclusion

In this paper, the problem of practical FTC based on event-triggered strategy in second-order multi-agent system is studied. Under the designed distributed event-triggered scheme, the practical FTC of the second-order system can be achieved. Compared with Qian[38], in this paper, the communication among the agents does not need continuous information exchange with the neighbors, not only in the control protocol, but also in the event-triggered function. The theoretical results are supported through a numerical example. Further work will study the effects of disturbances acting on individual nodes.

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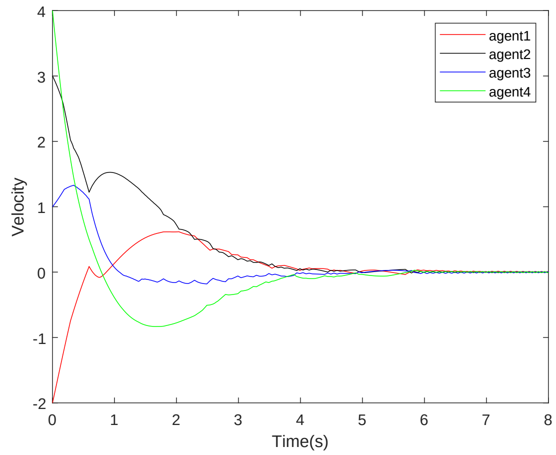

Fig. 3. The trajectories of all agents' velocity

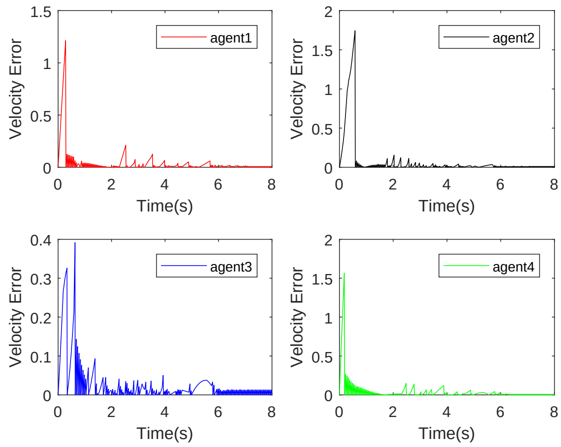

Fig. 5. Evolution of velocity errors of all agents

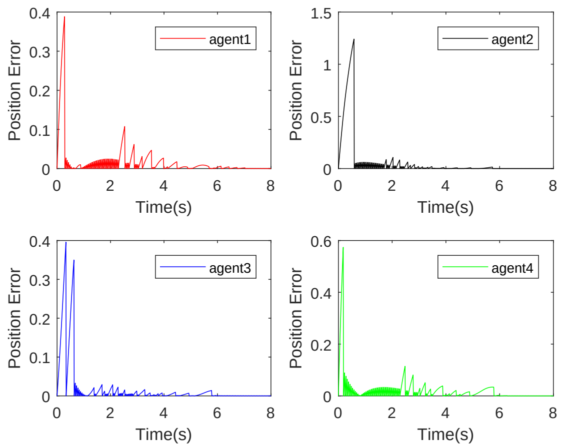

Fig. 4. Evolution of position errors of all agents

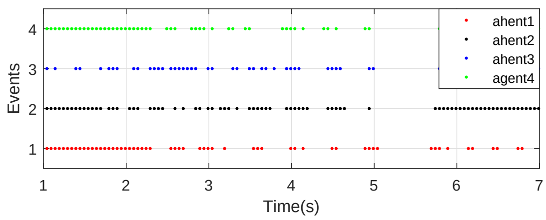

Fig. 6. Events of all agents

Elite Teacher Project.

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