

The Formation Transformation of the UUVs Based on the Leader-Follower Strategy and the Binary-Tree Structure

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1 **This paper addresses the approaches for the**
2 **formation transformation of the Unmanned**
3 **Underwater Vehicles (UUVs), which only rely on**
4 **sensor measurement. Due to the limited measuring**
5 **range, the leader-follower strategy and the binary-**
6 **tree structure are employed to design the desired**
7 **formation and the switching process. In order to**
8 **achieve the desired formation, the controller for**
9 **each follower UUV is designed with the integral**
10 **sliding mode method, which makes the follower**
11 **surge velocity and heading angle achieve consistent**
12 **with the leader UUV. The simulation results**
13 **demonstrate the formation transformation can be**
14 **achieved, and the approaches adopted in the paper**
15 **are effective.**

16
17 **Keywords:** Unmanned Underwater Vehicles,
18 Formation Transformation, Leader-Follower, Binary-
19 Tree and Integral sliding mode control

20 1. Introduction

21 With the improvement of the task complexity, it is
22 more and more difficult for a single agent to complete
23 the tasks. Multi-agent cooperation, however, can
24 increase the redundancy and robustness of the system
25 and improve the efficiency of task execution[1]. In this
26 case, the formation control attracted people's attention
27 and became an important study field of multi-agent
28 system, which requires that the agents can form the
29 desired formation.

30 The common strategies for formation control
31 include leader-follower strategy[2,3,4], behavior
32 based strategy, artificial potential field strategy[5,6]
33 and virtual structure strategy[7]. Among them, the
34 leader-follower strategy is very effective to realize the
35 distributed control. The common methods for tracking
36 control include sliding mode control[3,8,14],
37 backstepping control[9,10], event-triggered
38 strategy[15] and so on. Shikun Pang et al[2] designed
39 an inversion controller by using Lyapunov function, so
40 that the follower's trajectory can converge to the
41 reference path in a short time and the desired formation
42 can be obtained. However, the control law is relatively
43 complex and may be difficult to implement in
44 engineering. Zheping Yan et al[8] applied integral

45 sliding mode control to the trajectory tracking of the
46 underactuated UUV with parameter perturbation. Juan
47 Li et al[3] proposed a robust sliding mode formation
48 control method based on leader-follower strategy and
49 designed integral sliding mode surface for the velocity
50 error of the follower. However, the author only
51 illustrated the formation of three UUVs. As for the
52 formation of more UUVs, the vibration of the leaders
53 will be transmitted to the follower, which may
54 aggravate the vibration of the follower. Based on the
55 above literatures, this paper realizes the formation of 5
56 UUVs based on integral sliding mode control and PID
57 control. The follower UUV can track the leader UUV
58 with this method, and the appropriate parameters can
59 be designed to help reduce the vibration of the
60 follower UUV.

61 The above references mainly studied the formation
62 generation and maintenance. In practice, different
63 formations are needed to adapt to different
64 environments and finish different tasks, so the study of
65 formation transformation is significant. According to
66 the literature we investigated, most of the related
67 researches are focused on the UAV, and there are less
68 researches on the switching formation of UUV.
69 Daifeng Zhang et al[11] proposed a novel switching
70 formation method for quadrotor UAV based on
71 binary-tree, which can realize the transformation
72 between the herringbone-shape and the complete
73 binary-tree shape topology of 7 UAVs. With the use of
74 binary-tree, this method simplifies the rules of
75 formation transformation and saves computing costs.
76 However, this conference did not consider the
77 complicated aerodynamics of UAV, and the switching
78 process relied on the information of communication.
79 However, underwater communication is seriously
80 limited, unable to transmit a large amount of
81 information. Therefore, sensors are necessary in
82 underwater environment, which motivates our
83 research for UUV formation control with
84 measurement constraints. Therefore, the paper
85 introduced the hydrodynamic model of UUV and
86 binary-tree topology is adopted to formation
87 transformation of UUVs. Furthermore, herringbone-
88 shape formation is set as the standard formation.

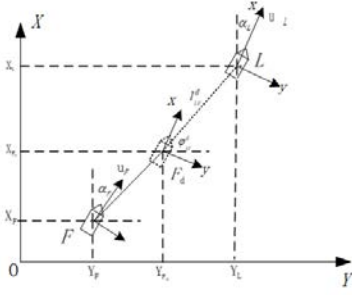


Fig. 1 The leader-follower model

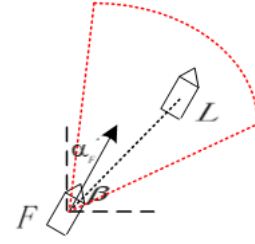


Fig. 2. The measuring range of the laser sensor

1 The main contributions in this paper are as follows:
 2 i)The detailed approaches are given to achieve
 3 formation transformation of 5 UUVs, which only rely
 4 on sensor measurement. The transformation among
 5 herringbone-shape and line-shape and spear-shape are
 6 considered in this paper. ii) This paper points out the
 7 problems of using the sliding mode tracking controller
 8 during the switching process, and puts forward some
 9 improved methods.

10 The contents of this paper are as follows: The
 11 model of the UUV, the measurement limitation and the
 12 binary-tree topology are described in Section 2. In
 13 Section 3, the control system of the follower UUV is
 14 introduced. The detailed formation transformation
 15 rules are introduced in Section 4. In Section 5, the
 16 simulation results are shown, and the paper is
 17 summarized in Section 6.

18 2. Problem Description

19 The model of the UUV and the binary-tree topology
 20 of the formation are introduced in this section. The
 21 section also includes the description of the limited
 22 measuring range.

23 2.1. The model of the UUV

24 **Figure. 1.** shows the leader-follower model of the
 25 UUVs, which is based on the model in reference [2].
 26 $V = (u, v, r)^T$, where (u, v) are the surge and sway
 27 velocities and r is the angular velocity of the UUV in
 28 the body coordinate system. $\mu = (X, Y, \alpha)^T$, where
 29 (X, Y) is the coordinate of the UUV in the Cartesian
 30 coordinate system, and the heading angle α is
 31 defined as the angle between the UUV orientation and
 32 the positive X axis of the coordinate system.

33 the kinematic model of the UUV is

$$\begin{aligned} \dot{X} &= u \cos \alpha - v \sin \alpha \\ \dot{Y} &= u \sin \alpha + v \cos \alpha. \\ \dot{\alpha} &= r \end{aligned} \quad (1)$$

37 The transformation matrix between the UUV body
 38 coordinate system and the cartesian coordinate system
 39 is defined as

$$R(\alpha) = \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (2)$$

41 The hydrodynamic model of UUV[12] is

$$\begin{cases} \dot{\mu} = R(\alpha)V \\ M\dot{V} = \tau - C(V)V - D(V)V \end{cases} \quad (3)$$

43 where $C(V)$ is a skew-symmetric matrix of Coriolis,
 44 and $D(V)$ is a nonlinear damping matrix. M is the
 45 system inertia matrix. The definition and parameter
 46 setting of $C(V)$, $D(V)$ and M can be referred to [12].
 47 The vector $\tau = (\tau_u, \tau_v, \tau_r)^T$ is the control force and
 48 the moment. Because the UUV considered is
 49 underactuated, $\tau_v = 0$.

50 2.2. The Measurement Limitation

51 The UUV considered has a laser sensor at the head.
 52 The follower UUV confirms its leader UUV and the
 53 relative position with the leader by using the
 54 measurement information. The measurement range of
 55 the laser sensor is limited. We set the measurement
 56 angle range of UUV to $\pm\theta$ and the farthest
 57 measurement distance to S_{max} , which are shown in
 58 **Fig. 2.**

59 2.3. The Topology of the Formation

60 In this paper, the directed graph $D = (P, E, A)$ is
 61 used to represent the formation topology. $P =$
 62 $\{p_1, p_2, \dots, p_N\}$ is the set of all UUVs and E is the set
 63 of all edges. Each edge is oriented from the leader to
 64 its follower. A is the adjacency matrix of the topology.

65 According to the leader-follower strategy and the
 66 constraints of the measurement, the formation that the
 67 UUVs can organize is designed as a tree structure with
 68 the main leader UUV as the root node.

69 The state of the follower UUV will not be fed back
 70 to its leader, so it is necessary to design the specific
 71 rules for each UUV to realize the formation
 72 transformation. To simplify the rules, the binary-tree
 73 structure is employed to design the formation topology
 74 and the switching process. Each node in the binary-
 75 tree can have at most two children. In other words,
 76 each leader in the formation has at most two followers.
 77 The subtree represented by the child node is called the
 78 left subtree or the right subtree[11], which is
 79 determined by the relative position of the follower to
 80 its leader. **Fig. 3** shows the herringbone-shape, the
 81 spear-shape and the line-shape formation of 5 UUVs
 82 designing by binary-tree.

83 3. The control System of the Follower UUV

84 In order to enable the follower UUV to track its
 85 leader, the control system of the follower UUV is
 86 designed as **Fig. 4.** The kinematic controller is

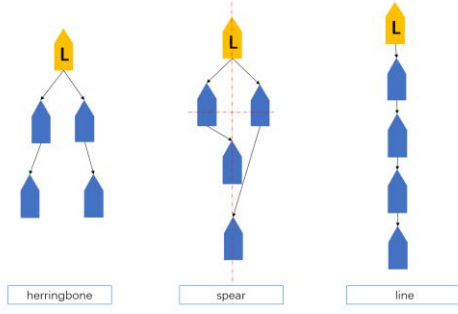


Fig. 3 The tree formations of the herringbone-shape, the spear-shape and the line-shape

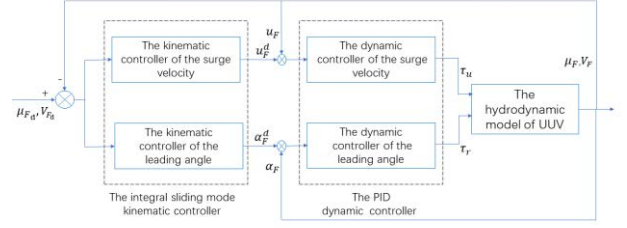


Fig. 4 Control system of the follower UUV

1 designed with the integral sliding mode control and the
2 leader-follower strategy. The PID method is applied in
3 the dynamic controller. F_d is the desired point of the
4 follower UUV.

5 The kinematic controller calculates the errors of μ
6 and V and adjusts the control laws of heading angle
7 α_F^d and surge velocity u_F^d . u_F^d and α_F^d are the
8 reference inputs of PID controller. The outputs of PID
9 controller adjust the control force and the moment
10 $\tau = (\tau_u, \tau_v, \tau_r)^T$ of UUV, so that UUV can update its
11 states. Then the states feed back to the kinematic
12 controller and PID dynamic controller.

13 3.1. The Leader-Follower model

14 The leader-follower model applied refers to the
15 formation model in reference[2], and the calculation is
16 simplify on the basis of it. The sketch map of the
17 leader-follower model is shown in **Fig. 1**. The distance
18 between the leader and the follower is denoted by l_{LF} ,
19 and the relative angle of sight is denoted by φ_{LF} . The
20 desired values of them are l_{LF}^d and φ_{LF}^d . $\mu_{F_d} =$
21 $(X_{F_d}, Y_{F_d}, \alpha_{F_d})^T$ can be calculated by

$$22 \begin{cases} Y_{F_d} = Y_L - l_{LF}^d \cos \varphi_{LF}^d \\ X_{F_d} = X_L - l_{LF}^d \sin \varphi_{LF}^d \\ \alpha_{F_d} = \alpha_L \end{cases} \quad (4)$$

23 $V_{F_d} = (u_{F_d}, v_{F_d}, r_{F_d})^T = V_L$, so V_L will replace
24 V_{F_d} for calculation in the following.

25 The position deviations between the follower UUV
26 and the leader UUV in the leader's body coordinate
27 system are denoted by $E_1 = (e_x, e_y)^T$, in which

$$28 \begin{cases} e_x = (Y_{F_d} - Y_F) \sin \alpha_L + (X_{F_d} - X_F) \cos \alpha_L \\ e_y = (Y_{F_d} - Y_F) \cos \alpha_L - (X_{F_d} - X_F) \sin \alpha_L \end{cases} \quad (5)$$

29 The deviations of the heading angle between the
30 leader UUV and the follower UUV is defined as

$$31 e_\alpha = \alpha_L - \alpha_F \quad (6)$$

32 3.2. The Design of the Kinematic Controller

33 With (2), (7) and (8) it can be obtained that
34
$$\begin{cases} \dot{e}_x = u_L - u_F \cos e_\alpha - v_F \sin e_\alpha + e_y \dot{\alpha}_L \\ \dot{e}_y = v_L + u_F \sin e_\alpha - v_F \cos e_\alpha - e_x \dot{\alpha}_L \\ \dot{e}_\alpha = \dot{\alpha}_L - \dot{\alpha}_F \end{cases} \quad (7)$$

35 The rotation matrix between the body coordinate
36 systems of the leader and the follower is defined as

$$37 R_1(\alpha) = \begin{bmatrix} \cos e_\alpha & -\sin e_\alpha \\ \sin e_\alpha & \cos e_\alpha \end{bmatrix} \quad (8)$$

38 The position deviations between the follower UUV
39 and the leader UUV in the follower's body coordinate
40 system is denoted by $E_2 = (\varepsilon_1, \varepsilon_2)^T$, and

$$41 E_2 = R_1(\alpha) \times E_1 \quad (9)$$

42 By deriving ε_1 it can be obtained that

$$43 \dot{\varepsilon}_1 = -\varepsilon_2 \dot{e}_\alpha + u_F + \rho_1 + \rho_2, \quad (10)$$

44 where

$$45 \begin{aligned} \rho_1 &= -\cos e_\alpha (e_y \dot{\alpha}_L + u_L), \\ \rho_2 &= \sin e_\alpha (v_L - e_x \dot{\alpha}_L). \end{aligned} \quad (11)$$

47 An integral sliding mode controller which can
48 eliminate the reaching process and thus improves
49 robustness is used to get the reference surge velocity
50 u_F^d . The first order sliding mode surface[3] is
51 designed as

$$52 S = \varepsilon_1 + k_1 \left(\int_0^t \varepsilon_1(\tau) d\tau + \varepsilon_1'(0) \right). \quad (12)$$

53 where $k_1 \in R_+$. $\varepsilon_1'(0)$ is the value of the integral
54 term when $t = 0$. We suppose

$$55 \varepsilon_1'(0) = -\frac{1}{k_1} \varepsilon_1(0), \quad (13)$$

56 so that $S = 0$ when $t = 0$. This shows the initial
57 state is on the sliding mode surface.

58 By deriving S we can get

$$59 \dot{S} = \dot{\varepsilon}_1 + k_1 \varepsilon_1. \quad (14)$$

60 If $\dot{S} = 0$, $S = 0$ at any time. By substituting (10)
61 into it, the desired value of surge velocity can be
62 calculated by

$$63 u_F^d = -k_1 \varepsilon_1 + \varepsilon_2 \dot{e}_\alpha - \rho_1 - \rho_2. \quad (15)$$

64 In order to ensure that the follower reaches the
65 desired point F_d , it's necessary to make the
66 orientation of the follower point to F_d . The angle
67 relation between the follower and F_d is shown in **Fig.**
68 **5**. The desired value of heading angle is

$$69 \alpha_F^d = \alpha_L + \arctan \left(\frac{e_y}{b} \right), \quad (16)$$

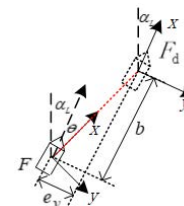


Fig. 5 The angle relation

1 where $b \in R_+$.

2 3.3. The Design of the Dynamic Controller

3 Two PID controllers, whose reference inputs are
4 u_F^d, α_F^d , are designed to get the control force and the
5 moment $\tau = (\tau_u, \tau_v, \tau_r)^T$ of UUV,

$$6 \begin{cases} \tau_u \propto K_{p_1}((u_F^d - u_F) + \frac{1}{T_{i_1}} \int_0^t (u_F^d - u_F) dt + T_{D_1} \frac{d(u_F^d - u_F)}{dt}) \\ \tau_v = 0 \\ \tau_r \propto K_{p_2}((\alpha_F^d - \alpha_F) + \frac{1}{T_{i_2}} \int_0^t (\alpha_F^d - \alpha_F) dt + T_{D_2} \frac{d(\alpha_F^d - \alpha_F)}{dt}) \end{cases}, \quad (17)$$

7 where $K_{p_1}, K_{p_2}, K_{p_3}, T_{i_1}, T_{i_2}, T_{D_1}, T_{D_2} \in R, T_{i_1}, T_{i_2}, T_{D_1}, T_{D_2} \neq 0$.
8 Equation (17) shows that the outputs of PID controller
9 adjust the values of τ . The maximum values are
10 determined by the structure of UUV and the actuator
11 configuration. The paper sets $|\tau_u| \leq 2.0N, |\tau_r| \leq$
12 $2.0N \cdot m$.

13 3.4. Stability Analysis

14 We define the following Lyapunov function.

$$15 V_{Lya} = \frac{1}{2}(\varepsilon_1^2 + \varepsilon_2^2). \quad (18)$$

16 With the control law (16), the orientation of the
17 follower point to F_d , so $\varepsilon_2 = 0$.

18 By deriving formula (18) we can get

$$19 \begin{aligned} \dot{V}_{Lya} &= \varepsilon_1 \dot{\varepsilon}_1 + \varepsilon_2 \dot{\varepsilon}_2 \\ &= -k_1 \varepsilon_1^2 + 0 \leq 0. \end{aligned} \quad (19)$$

20 ε_1 can converge to 0.

21 Thanks to the wide range of applications and good
22 performance of PID method, the stability analysis of
23 the errors of α_F and u_F is omitted.
24

25 4. The Formation Transformation Based on 26 the Binary-Tree

27 This section describes the design of the formation
28 transformation process based on the binary-tree, and
29 the controller introduced in Section 3 is used to adjust
30 the relative position of the follower UUV to its leader.
31 At the same time, the problems of using the sliding
32 mode tracking controller during the switching process
33 are pointed, and some corresponding improved
34 method are put forward.

35 Before designing the transformation process, it is
36 needed to confirm the left and right subtrees of the
37 main leader UUV p_1 . For the follower UUV p_i of
38 p_1 ,

$$39 \begin{cases} \varphi_{LF}^d + \alpha_L \leq \frac{\pi}{2}, & p_i \text{ and its followers belong to the left subtree} \\ \varphi_{LF}^d + \alpha_L > \frac{\pi}{2}, & p_i \text{ and its followers belong to the right subtree} \end{cases} \quad (20)$$

40 The UUVs of the left and the right subtrees follow
41 different transformation rules.

42 In order to simplify the rules, this paper sets the
43 herringbone formation as the standard formation. The
44 herringbone formation can be directly converted into
45 the line and the spear. The conversion between the line
46 formation and the spear needs to be mediated by the
47 herringbone formation.

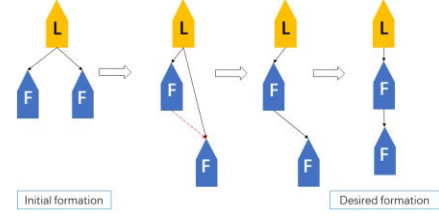


Fig. 6 Formation transformation of the 3 UUVs

48 4.1. The transformation process of 3 UUVs

49 In the process of the transformation, there are two
50 operations needed, which are adjusting the position of
51 the follower UUV relative to the leader and changing
52 the leader of the follower UUV. The group of 3 UUVs
53 is the smallest unit that can realize the above two
54 operations. The transformation process of 3 UUVs
55 from herringbone-shape to line-shape is shown in Fig.
56 6.

57 The rules summarized from Fig. 6 are as follow.

58 The rules for the left-subtree node: adjust φ_{LF}^d so
59 that $\varphi_{LF}^d + \alpha_L = \frac{\pi}{2}$ (the follower goes forward in the
60 line-shape with the leader).

61 The rules for the right-subtree nodes:

- 62 • Step 1: Stay away from the leader so that it can
63 detect both the leader and the left subtree node
64 at the same time.
- 65 • Step 2: Take the left subtree node as the leader,
66 then adjust φ_{LF}^d so that $\varphi_{LF}^d + \alpha_L = \frac{\pi}{2}$.

67 4.2. The transformation process of 5 UUVs

68 Based on the formation transformation rules of 3
69 UUVs, the formation transformation process of more
70 UUVs is designed as follow. The UUV in the topology
71 are layered firstly. The main leader UUV p_1 belongs
72 to layer 0. If one UUV belongs to layer m ($m > 0$), its
73 leader belongs to layer $m-1$. In the herringbone
74 formation in Fig. 3, there are two nodes in layer m
75 ($m > 0$). They belong to the left subtree and the right
76 subtree respectively and are recorded as p_{left}^m and
77 p_{right}^m . Suppose there are N_l nodes in the left subtree
78 and N_r nodes in the right subtree of p_1 , the
79 transformation rules between the line and the
80 herringbone formation are introduced as follow.

81 The rules for the left subtree nodes: go forward in
82 the line-shape with the leader and shorten the distance
83 with the leader.

84 The rules for the right subtree nodes:

- 85 • Step 1: p_{right}^1 stay away from p_1 until it can
86 detect both p_1 and p_{left}^1 at the same time. Other
87 nodes of the right subtree go forward in the line-
88 shape with p_{right}^1 .

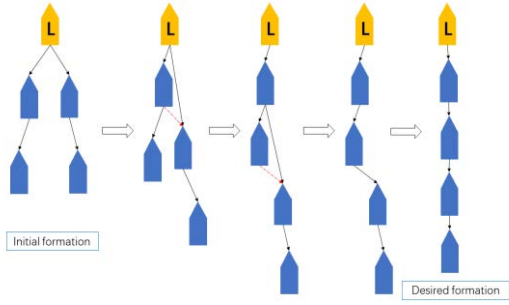


Fig. 7 Formation transformation of the 3 UUVs

- 1 • Step 2: p_{right}^1 takes p_{left}^1 as its leader, then stay
- 2 away from p_{left}^1 until it can detect both p_{left}^1
- 3 and p_{left}^2 at the same time.
- 4 • Step 3: p_{right}^1 takes p_{left}^2 as its leader, then go
- 5 forward in the line-shape. Finally, p_{right}^1 takes
- 6 p_{left}^N as its leader.

7 The transformation process of 5 UUVs from the
8 herringbone to the line formation resorting to the
9 above rules is shown in Fig. 7. The rules opposite to
10 the above can realize the transformation from the line
11 formation to the herringbone, which is deferent from
12 reference [11]. p_{left}^1 keep moving forward and finally
13 takes p_1 as the leader. Other UUVs return to their
14 positions in the herringbone formation.

15 Conversion between the herringbone formation and
16 the spear formation does not require the UUVs change
17 their leaders. Only the positions of the last layer nodes
18 relative to first layer nodes need to be adjusted. The
19 desired spear formation is shown in Fig. 3.

20 4.3. The improvement of the rules

21 Because of the application of the sliding mode
22 method in the follower UUV's kinematic controller,
23 the big change of the reference input will lead to the
24 output oscillation and even the output divergence.
25 Therefore, some improvement of the rules are
26 proposed to help reduce the output oscillation.

- 27 • When the follower UUV moves forward/
28 backward for a long distance relative to its
29 leader, the desired value of the relative distance
30 l_{LF}^d increases/decreases linearly, instead of the
31 large step input to the kinematic controller.
- 32 • Considering the cascade structure of the UUVs,
33 the vibration of the leader UUV's stage will
34 affect the follower. So it is necessary to wait for
35 a period of time before executing the next rule
36 in the transformation process, so that the
37 follower has enough time to reach the stable
38 state.

39 5. Simulation Results

40 Some parameters of the desired formation of 5
41 UUVs in the simulation are given as Table 1.

Table. 1 Parameter setting.

	herringbone		line		cross	
	l_{LF}^d	l_{LF}^d	l_{LF}^d	φ_{LF}^d	l_{LF}^d	φ_{LF}^d
p_{left}^1	5.0m	$\pi/5$	2.0m	$\pi/2 - \alpha_L$	5.0m	$\pi/5$
p_{right}^1	5.0m	$\pi/3$	2.0m	$-\alpha_L$	5.0m	$\pi/3$
p_{left}^2	3.0m	$\pi/5$	3.0m	$\pi/2 - \alpha_L$	3.0m	$\pi/3$
p_{right}^2	3.0m	$\pi/3$	2.0m	$\pi/2 - \alpha_L$	7.0m	$\pi/5$

42 The leader's preset trajectory is a line with a slope
43 of 1. The initial values of V and μ of the leader and
44 the followers are given as follow: $V_L = V_F^l = V_F^r =$
45 $(0,0,0)^T$; $\mu_L = (40,0,0)^T$, $\mu_{left}^1 = (35, -3,0)^T$,
46 $\mu_{right}^1 = (37, -5,0)^T$, $\mu_{left}^2 = (30, -6,0)^T$,
47 $\mu_{right}^2 = (34, -10,0)^T$.

48 5.1. The Formation of the Herringbone

49 Figure. 8 shows the trajectories of 5 UUVs when
50 forming the herringbone. There is no intersection
51 among the tracks, which means there is no collision
52 among the UUVs. No UUV dropping out also
53 indicates that the follower UUV and its leader meet the
54 measurement constraints.

55 Figure. 9 show the surge velocity and leading angle
56 curves of 5 UUVs. It can be seen from the figure that
57

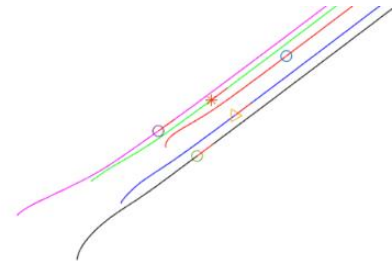


Fig. 8 The Herringbone formation of 5 UUVs

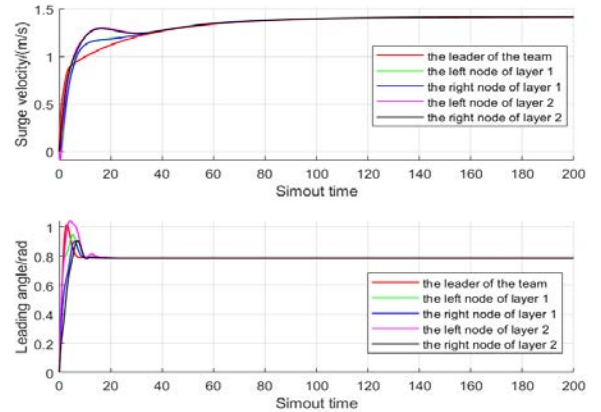


Fig. 9 Surge velocity and leading angle curves of the 5 UUVs

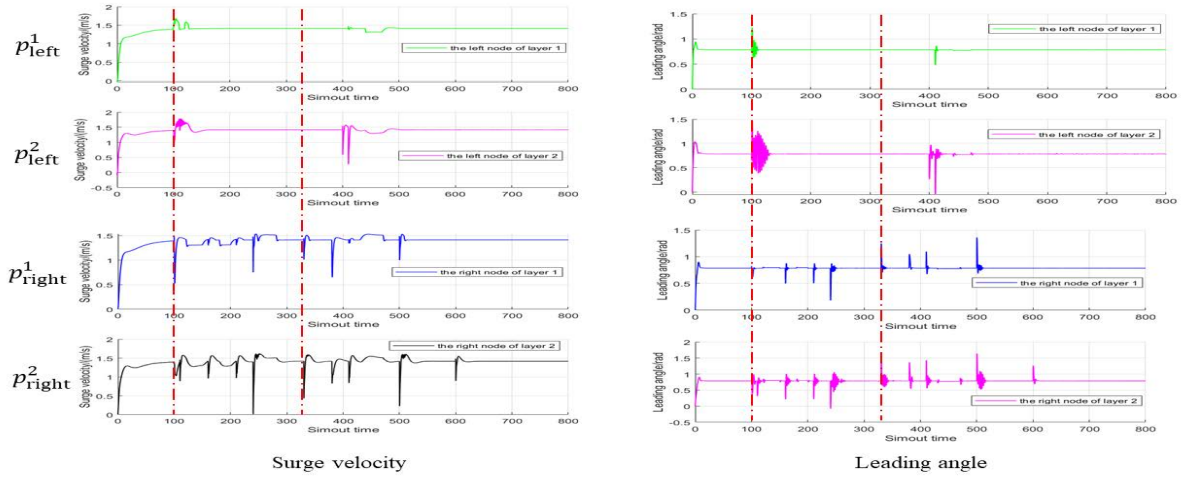


Fig. 11 The surge velocity and the leading angle curves of 4 follower UUVs

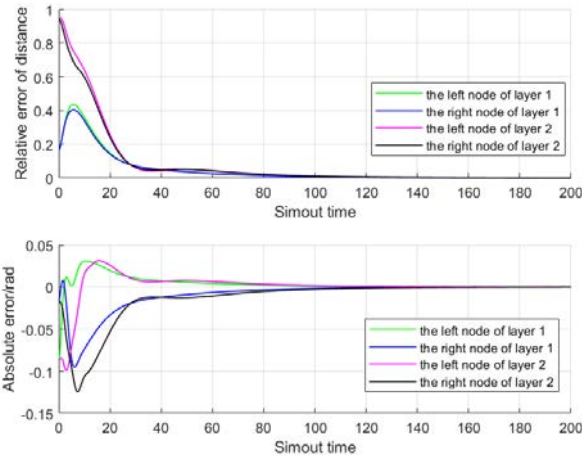


Fig. 10 The formation error of the follower UUVs

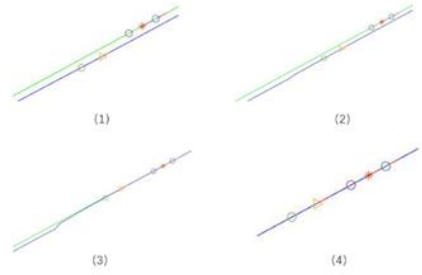


Fig. 12 the formation transformation process from the herringbone to the line. (1) The left subtree nodes keep straight ahead with the main leader of the team, and the right subtree nodes close to the main leader's track. p_{right}^1 changes its leader to p_{left}^1 . (2) p_{right}^1 changes its leader to p_{left}^2 . (3) p_{right}^1 and p_{right}^2 go straight ahead with p_{left}^2 . (4) Adjust the relative distance between nodes to achieve the desired straight line formation

1 the velocities and the forward directions of the 5 UUVs
 2 can be consistent finally.
 3 **Figure. 10** shows the relative error of the relative
 4 distance l_{LF} and the absolute error of the relative angle
 5 of sight φ_{LF} . The figure proves the follower UUVs can
 6 reach the desire point in the formation.

7 **5.2. The Transformation process between the**
 8 **Herringbone and the Line Formation**

9 According to the formation transformation rules
 10 designed in Section 4, the simulation is carried out and
 11 the results are shown as follow.

12 **Figure. 11** show the surge velocity and the leading
 13 angle of 4 follower UUVs. The first red dotted line
 14 indicates the starting time of the transformation
 15 process from the herringbone to the line, and the
 16 second indicates the starting time of the transformation
 17 process from the line to the herringbone. The
 18 oscillation of p_{left}^1 and p_{right}^1 will make p_{left}^2 and
 19 p_{right}^2 produce more violent oscillation, which is the
 20 problem of applying the sliding mode method. p_{right}^1
 21 and p_{right}^2 employ the improved method introduced
 22 in Section 4 while p_{left}^1 and p_{left}^2 do not apply it. It
 23 can be seen from the results that the oscillation

24 frequency and the amplitude of the heading angle of
 25 p_{right}^2 is smaller than that of p_{left}^2 , which shows that
 26 the improved method is effective.

27 The mutation of the values shown in **Fig. 11**
 28 corresponds to the four processes shown in **Fig. 12**.
 29 The velocities and the leading angles of the 5 UUVs
 30 can be consistent finally.

31 In the transformation process between herringbone
 32 and cross, only the UUVs of layer 2 is needed to
 33 change the relative position with their leader. The
 34 cross formation is shown in **Fig. 13**.

35 **Figure. 14** show the surge velocity curves and the
 36 leading angle curves of p_{left}^2 and p_{right}^2 .

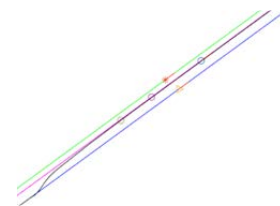


Fig. 13 The cross formation of 5 UUVs. The purple circle is p_{left}^2 and the green circle is p_{right}^2

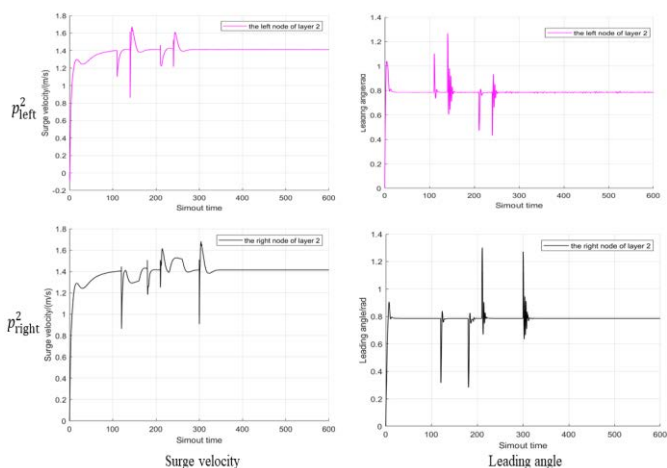


Fig. 14 the surge velocity curves and the leading angle curves of p_{left}^2 and p_{right}^2

1 **6. Conclusion**

2 In this paper, the rules of formation transformation
 3 based on leader-follower strategy are designed for
 4 UUVs with measurement constraints. The formation
 5 controller for follower UUV is designed with integral
 6 sliding mode control and some other calculation
 7 methods. In this way, the follower can reach the
 8 desired position in the formation and move forward at
 9 the same speed and direction as the leader. The
 10 simulation results show the UUVs can achieve the
 11 formation and verify the effectiveness of the methods
 12 employed in the paper. The methods designed in this
 13 paper are applicable to a small amount of UUVs. In
 14 future work, the authors will try to expand the
 15 formation scale and design formation transformation
 16 rules in reference to the bionic mechanism (such as the
 17 movement of fish) to get the more applicable topology
 18 model.

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