Paper:

Terminal Angle Constraint Guidance Law through Time-to-go Estimation Using Neural Network

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Abstract. In this paper, a terminal guidance law for the impact-angle and the terminal angle of attack (AoA) constraint is proposed. The main feature of this guidance law is that the neural network is utilized to fit the relationship between flight states and time-to-go, and the information of altitude is used in guidance law to achieve terminal-angle constraint. The algorithm firstly is designed using the SDRE method to constrain the impact-angle and the terminal AoA using the function of altitude and time-to-go respectively. Then, the neural network is trained to achieve the impact-angle constraint and the terminal AoA constraint simultaneously using the information of altitude. The simulation results show that the proposed guidance law is effective for terminal angle constraining and a comparative study is carried out to show that it has shorter computing time, flatter trajectories, and faster convergence rate of AoA than the SDRE guidance law designed by the function of time-to-go.

Keywords: neural network, time-to-go estimation, terminal guidance, impact-angle constraint, angle of attack constraint

1. Introduction

In modern warfare, the terminal requires not only the precision of hitting the target, but also the constraint of the collision angle, the collision time and the impact velocity. In order to enhance the damaging effect of collision, Kim and Grider first introduced the impact-angle constraint in the terminal guidance problem in 1973[1]. Since then, the impact-angle constraint attracted considerable attention.

In terms of impact-angle constraint, the optimal and suboptimal control method, the sliding mode variable structure guidance method and the improved proportional guidance method have been studied more systematically. Along with the development of modern control theory, the control methods based on the optimal and suboptimal were widely used in the guidance with impact-angle constraint. Ryoo utilized the linear quadratic optimal control theory for the linearized kinematics model to constrain impact-angle and obtain the analytical solution of the optimal energy[2]. In the following year, the value function with time-to-go was used to further analyze the above problem[3]. Generalized weighted optimal guidance laws with impact-angle constraints were studied for first-order lag control systems and lag-free control systems respectively using Schwarz's inequality approach by Zhang et al.[4]. Aiming at the problem of fixed target missile guidance with virtual hand-over point, a global energy optimal guidance law with/without terminal angle constraint was designed based on the optimization theory in Hilbert space by Li et al.[5]. Ratnoo first applied the state-dependent Riccati equation (SDRE) within the landing constraint guidance community, and designed a suboptimal guidance law to realize the terminal impact-angle constraint[6]. While the impact-angle constraint has been guaranteed by the optimal guidance law, several scientists also added other constraints. Reference [7] restricted the acceleration effectively to avoid guidance instruction saturation, leaving a certain control margin for the suppression of external interference. In terms of an efficient salvo attack of antiship missiles or a cooperative mission of unmanned aerial vehicles (UAVs), the reference [8] achieved not only the desired impact-angle but also the impact time simultaneously. However, the above studies all used time-to-go in the guidance laws. When the guidance trajectory is relatively curved, t_{go} is often difficult to estimate accurately. Zhao proposed to replace t_{go} with the accurately acquired altitude-to-go to design the guidance law with terminal angle constraints[9].

Based on hitting the target with a certain impact-angle, the difference between the terminal pitching angle and impact-angle is expected to approach zero. In other words, the terminal AoA is expected to approach zero so as to improve the damaging effect of the warhead. Some researchers have conducted in-depth studies on the terminal AoA constraint. York directly assumed that the AoA was small and ignored in the reentry guidance study[10], which was equivalent to directly "guaranteeing" the terminal AoA constraint. The main disadvantage of this assumption is that, the AoA can not always be small especially when the trajectory is relatively curved. The optimal guidance law derived by Sun achieved zero AoA hitting by and large, which was of great significance for the penetration attack of air-to-ground missiles [11]. The relatively large terminal AoA under certain conditions remained a major challenge. Rusnak clarified the AoA and its dynamic characteristics in the derivation of the guidance law and realized the terminal angle constraint by changing the AoA but he did not ensure that the terminal AoA approached zero at the same time[12]. Xing and Chen introduced the AoA into the state space equation, and controlled impact-angle and the AoA respectively, so as to realize the terminal angle constraint[13]. A major problem with this method was that the model required a large amount of calculation by using the fourth-order matrix to solve the problem. On the basis of the previous work, Zhang et al. deduced a new terminal guidance law by using the linear optimal control method, and its formula was simpler than the former, which realized the impact-angle constraint and the terminal AoA approaching zero[14]. Varma and Parwana introduced the kinematic equations of the missile body's rotation around the center of mass, which not only achieved the impact-angle control command, but also ensured the missile's pitch angle control command, thus improved the effectiveness of the collision[15, 16].

Inspired by Zhao [9] and Parwana [16], two SDRE controllers are designed to achieve expected terminal attitude and impact-angle, which are respectively an impact-angle constraint controller using altitude-to-go information and an AoA constraint controller using time-to-go information derived from the neural network algorithm. Because the inputs of the neural network are altitude information and other states in real-time, time-to-go information is not needed during the whole guidance process. The contributions of this paper can be summarized as follows. The terminal AoA constraint is considered in this paper but not analyzed in [9]. The information of altitude directly measured is used and two second-order SDREs derived from the neural network are designed to obtain guidance law in this paper, while the estimation of the time-to-go is used to get guidance law and the fourth-order online SDRE were solved in [16]. In addition, two second-order SDREs derived to get analytical solutions greatly reduce the trajectory calculating time. This method can avoid calculating t_{go} directly and realize the zero AoA collision effectively, thus improving the damaging effect of the missile attack department greatly.

The rest of the paper is organized as follows. Guidance objectives, SDRE guidance laws and the guidance law structure which is the main idea of this paper are given in Section 2. The neural network task which describes the designing process of the network in details is provided in Section 3. Simulation results and discussion of comparisons of the different algorithms are provided in Section 4. Conclusions and possible future work are discussed in the last section.

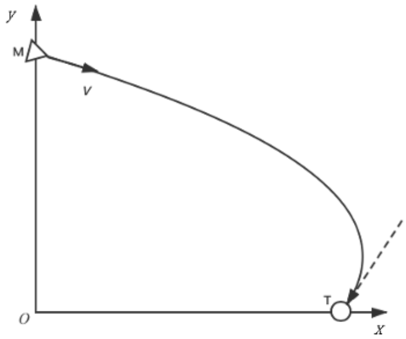

Fig. 1. Guidance trajectory

2. Guidance Law Structure

2.1. Guidance Problem Description

Consider an impact-angle-constrained scenario as shown in **Fig. 1**. The missile M is required to attack the stationary target T with expected impact-angle and zero AoA. The following model shown as Eq.(1) is used to describe the motion of the center of mass of the missile under 2-D conditions.

$$
\begin{aligned}
\dot{x} &= V\cos\gamma \\
\dot{y} &= V\sin\gamma \\
\dot{V} &= -g\sin\gamma - \frac{D}{m} \\
\dot{\gamma} &= \frac{-g\cos\gamma}{V} + \frac{L\cos\mu}{mV} \\
\dot{\vartheta} &= \omega_z \\
\dot{\omega}_z &= \frac{M_z}{I_{zz}}
\end{aligned} \qquad (1)
$$

where the normal acceleration can be defined as $a_y = \frac{L\cos\mu}{m}$, V indicates the velocity of the missile, γ indicates the elevation angle, ϑ indicates the pitching angle, g indicates gravity acceleration, m indicates the mass, D and L indicate respectively the drag force and lift force of the missile, μ indicates heeling angle, ω_z indicates the pitching angular velocity, M_z indicates the pitching moment and I_{zz} indicates the rotational inertia along the z axis. Therefore, the terminal angle constraint is accomplished by controlling the normal acceleration a_y and rotational torque M_z. The corresponding model is shown as Eq.(2).

$$
\begin{aligned}
\dot{x} &= V\cos\gamma \\
\dot{y} &= V\sin\gamma \\
\dot{V} &= -g\sin\gamma - \frac{D}{m} \\
\dot{\gamma} &= \frac{a_y - g\cos\gamma}{V} \\
\dot{\vartheta} &= \omega_z \\
\dot{\omega}_z &= \frac{M_z}{I_{zz}}
\end{aligned} \qquad (2)
$$

The goal in this paper is to constrain the impact-angle and the terminal AoA on the premise of accurately hitting the stationary ground target, which can be described as

$$\begin{aligned}&\lim_{t_{go}\to 0} x = x_f\\&\lim_{t_{go}\to 0} y = y_f\\&\lim_{t_{go}\to 0} \gamma = \gamma_f\\&\lim_{t_{go}\to 0} \vartheta = \vartheta_f = \gamma_f(\lim_{t_{go}\to 0} \alpha = 0)\end{aligned} \quad (3)$$

In order to avoid time-to-go information that needs to be evaluated, we use the following equations as the design objective.

$$\begin{aligned}&\lim_{y\to y_f} x = x_f\\&\lim_{y\to y_f} \gamma = \gamma_f\\&\lim_{y\to y_f} \vartheta = \vartheta_f = \gamma_f(\lim_{y\to y_f} \alpha = 0)\end{aligned} \quad (4)$$

where the subscript f indicates the expected value.

2.2. SDRE Guidance Law Designment

2.2.1. Guidance Law Designment with Impact-Angle Constraint

A new variable is defined as $Y = y_0 - y$, where y_0 indicates initial altitude of the missile. Then,

$$\begin{aligned}&\sigma_1 = x - x_f + \cos\gamma_f(Y - Y_f)\\&\sigma_2 = \frac{d\sigma_1}{dY} = \frac{dx}{dY} + \cot\gamma_f = -cot\gamma + \cot\gamma_f\end{aligned} \quad (5)$$

and the differential of σ_2 can be defined as

$$\frac{d\sigma_2}{dY} = \frac{1}{\sin^2\gamma}\gamma' \quad (6)$$

where $\gamma' = \frac{d\gamma}{dY}$ is seen as the auxiliary control input and state variables are defined as $\boldsymbol{X}_1 = [\sigma_1, \sigma_2]^T$. Then, the state space equation is shown as follows,

$$\dot{\boldsymbol{X}}_1 = \boldsymbol{A}_1\boldsymbol{X}_1 + \boldsymbol{B}_1\gamma' \quad (7)$$

where $\boldsymbol{A}_1 = \begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}, \boldsymbol{B}_1 = \begin{bmatrix}0\\\frac{1}{\sin^2\gamma}\end{bmatrix}$. Considering the following cost function,

$$J_1 = \frac{1}{2}\int_{Y_0}^{\infty}(\boldsymbol{X}_1^T\boldsymbol{Q}\boldsymbol{X}_1 + \boldsymbol{B}_1\gamma'^2)dY \quad (8)$$

where $\boldsymbol{Q}_1 = \begin{bmatrix}q_{11}^2 & 0\\0 & q_{12}^2\end{bmatrix}$ and $R_1 = 1$.

The relationship between the auxiliary variable γ' and the actual control input a_y is shown as follows,

$$\gamma' = \frac{d\gamma}{dY} = -\frac{a_y}{V^2\sin\gamma} \quad (9)$$

then,

$$a_y = -V^2\sin\gamma\gamma' \quad (10)$$

We can make that

$$q_{11} = q_{12}^2 = \frac{N_1}{(y-y_f)^2} \quad (11)$$

The asymptotic stability conditions of the system are verified as follows according to [17].

1. The $\{\boldsymbol{A},\boldsymbol{B}\}$ should be pointwise controllable in the domain of interest.

$$|\{\boldsymbol{B}_1, \boldsymbol{A}_1\boldsymbol{B}_1\}| = -\frac{1}{\sin^4\gamma} \neq 0$$

2. The function $f(\boldsymbol{X}) = \boldsymbol{A}(\boldsymbol{X})\boldsymbol{X} \in C^1$.

$$\boldsymbol{A}_1(\boldsymbol{X}_1)\boldsymbol{X}_1 = \begin{bmatrix}\sigma_2\\0\end{bmatrix} \in C^1$$

3. The initial condition must be satisfied $f(\boldsymbol{0}) = \boldsymbol{0}$.

$$f(\boldsymbol{0}) = \begin{bmatrix}0\\0\end{bmatrix}$$

4. The matrix $\boldsymbol{B} \neq 0$.

$$\boldsymbol{B}_1 = \begin{bmatrix}0\\\frac{1}{\sin^2\gamma}\end{bmatrix} \neq 0$$

5. That the matrix $\boldsymbol{Q} \geq 0$ and the matrix $\boldsymbol{R} > 0$ is satisfied.

The nonlinear feedback controller $\boldsymbol{U} = -\boldsymbol{R}^{-1}\boldsymbol{B}^T\boldsymbol{P}\boldsymbol{X}$ can be constructed after obtaining the matrix $\boldsymbol{P}$ according to the Riccati equation $\boldsymbol{A}^T\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A} - \boldsymbol{P}\boldsymbol{B}\boldsymbol{R}^{-1}\boldsymbol{B}^T\boldsymbol{P} + \boldsymbol{Q} = \boldsymbol{0}$. Thus, the analytic solution is derived as,

$$\begin{aligned}&a_y = g\cos(\gamma - \theta_n) +\\&V^2\sin(\gamma-\theta_n)\{\frac{N_1}{(y-y_f)^2}[x - x_f + \cot\gamma_f(y_f - y)] +\\&\sqrt{\frac{N_1(2\sin^2\gamma+1)}{(y-y_f)^2}}(-\cot\gamma + \cot\gamma_f)\}\end{aligned} \quad (12)$$

The coordinate system is rotated counterclockwise θ_n in order to solve the singular problem when the expected impact-angle is $0deg$, and the angle and displacement are replaced by the angle-of-sight and the distance between the missile and the target. The specific details can be seen in [9].

2.2.2. Guidance Law Designment with the Terminal AoA Constraint

Choose state variables as $\boldsymbol{X}_2 = [\vartheta, \omega_z]^T$ and M_z is control input, then state space equations are shown as follows,

$$\dot{\boldsymbol{X}}_2 = \boldsymbol{A}_2\boldsymbol{X}_2 + \boldsymbol{B}_2M_z \quad (13)$$

where $\boldsymbol{A}_2 = \begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}$ and $\boldsymbol{B}_2 = \begin{bmatrix}0\\\frac{1}{I_{zz}}\end{bmatrix}$. The corresponding cost function is shown as follows,

$$J_2 = \frac{1}{2}\int_{t_0}^{\infty}(\boldsymbol{X}_2^T\boldsymbol{Q}_2\boldsymbol{X}_2 + R_2M_z^2)dt \quad (14)$$

where $\boldsymbol{Q}_2 = \begin{bmatrix} q_{21}^2 & 0 \\ 0 & q_{22}^2 \end{bmatrix}$ and $R_1 = 1$. We can make that $q_{21} = (\frac{N_2}{t_{go}})^2$ and $q_{22} = 0$ where N_2 is constant.

Also, the asymptotic stability conditions of the system are verified as follows.

1. The $\{\boldsymbol{A}, \boldsymbol{B}\}$ should be pointwise controllable in the domain of interest.

$$|\{\boldsymbol{B}_2, \boldsymbol{A}_2\boldsymbol{B}_2\}| = -\frac{1}{I_{zz}^2} \neq 0$$

2. The function $f(\boldsymbol{X}) = \boldsymbol{A}(\boldsymbol{X})\boldsymbol{X} \in C^1$.

$$\boldsymbol{A}_2(\boldsymbol{X}_2)\boldsymbol{X}_2 = \begin{bmatrix} \omega_z \\ 0 \end{bmatrix} \in C^1$$

3. The initial condition must be satisfied $f(\boldsymbol{0}) = \boldsymbol{0}$.

$$f(\boldsymbol{0}) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

4. The matrix $\boldsymbol{B} \neq 0$.

$$\boldsymbol{B}_2 = \begin{bmatrix} 0 \\ I_{zz} \end{bmatrix} \neq 0$$

5. That the matrix $\boldsymbol{Q} \geq 0$ and the matrix $\boldsymbol{R} > 0$ is satisfied.

According to the Riccati equation, we can get

$$\boldsymbol{P} = \begin{bmatrix} q_{21}\sqrt{2q_{21}I_{zz}} & q_{21}I_{zz} \\ q_{21}I_{zz} & I_{zz}\sqrt{2q_{21}I_{zz}} \end{bmatrix} \quad . \ . \ . \ . \ . \ (15)$$

then the control input can be derived as follows,

$$M_z = -(\frac{N_2}{t_{go}})^2\vartheta - \frac{N_2}{t_{go}}\sqrt{2I_{zz}}\omega_z \quad . \ . \ . \ . \ . \ . \ . \ (16)$$

The control input is shown as follows in order to widen the terminal attitude angle,

$$M_z = -(\frac{N_2}{t_{go}})^2(\vartheta - \vartheta_f) - \frac{N_2}{t_{go}}\sqrt{2I_{zz}}\omega_z \quad . \ . \ . \ . \ (17)$$

2.3. Guidance Law Structure Construction

It can be seen that the missile can hit the target at expected impact-angle and attitude from the above derivation in theory. However, the control input M_z is also a function of t_{go}. In order to guide the missile by using the altitude information in real-time rather than t_{go}, we choose the neural network to fit the relationship between time-to-go and altitude. The specific details can be seen in Section 3.

Because the altitude information of the missile can be measured in real-time and at present, the guidance law is the function of the altitude, the structure of which can be constructed as shown in **Fig. 2**. The whole process of the algorithm flow is achieved as shown in **Fig. 3**. Firstly, SDRE guidance law with the impact-angle constraint using the information of y is designed. Then SDRE guidance law with the terminal AoA constraint using the information of t_{go} is also designed. Next, fit the relationship between y and t_{go} using the neural network algorithm. At last, the neural network model is added to achieve the impact-angle constraint and the terminal AoA constraint using the information of y.

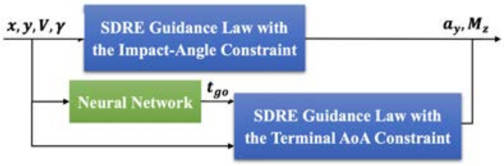

Fig. 2. Guidance law structure

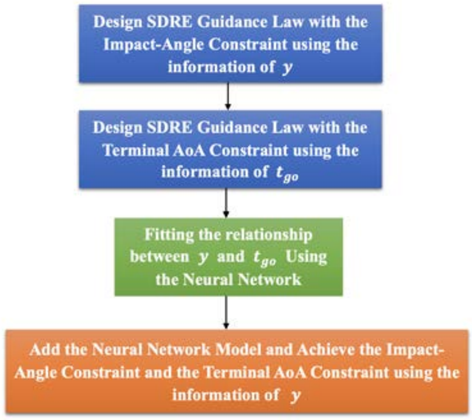

Fig. 3. Guidance law design flowsheet

3. Neural Network Task

3.1. Determining Inputs and Outputs

From **Fig. 2**, it can be seen that the neural network task is to output the information of t_{go} to the attitude controller, that is the SDRE guidance law with the terminal AoA constraint, so that the terminal AoA can converge to zero. Thus, the output of the neural network is set as t_{go}. Next, the inputs of the neural network need to be determined. The states of the missile include the altitude H (the same as y in this paper), the velocity V, the elevation angle γ of the missile and the horizontal range X between the missile and the target. Thus, the inputs of the neural network are set as $[H, V, \gamma, X]^T$. It can be seen that the number of features is only four, so it is enough to choose a neural network with three layers that includes one input layer, one hidden layer and one output layer. And as a rule of thumb, the number of neurons in the hidden layer is a multiple of the number of features, which is set as 64. The structure of the neural network is shown in **Fig. 4**.

3.2. Access to Training Data

Accurate training samples must be used to train the neural network model. Therefore, we choose the guidance law that only considers impact-angle constraint, namely the guidance law designment with impact-angle constraint in Section 2, to obtain training data. Because this guidance law is a function of altitude, the information of the time is not required during the whole process. For each trajectory simulated, there is an overall flight time T.

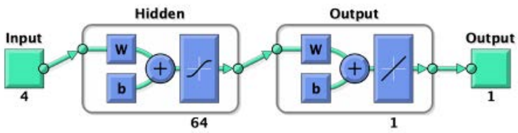

Fig. 4. Neural network structure

Table 1. Simulation conditions

Parameters	Values
Initial velocity V_0	$600m/s$
Initial position of the missile (x_0, y_0)	$(0, 3200)m$
Initial elevation angle γ_0	$-15deg$
Initial pitching angle ϑ_0	$-5deg$
Target position (x_f, y_f)	$(10000, 0)m$
Expected impact-angle γ_f	$0 : -10 : -150deg$
Limit of the normal acceleration $a_{y_{limit}}$	$\pm 15g$
Guidance coefficient N_1	10
Guidance coefficient N_2	3

During the flight, each state point $[x, y, V, \gamma]^T$ changes with time t, so a series of training samples can be obtained.

The inputs of the neural network are defined as $H = y$, $V = V$, $\gamma = \gamma$, $X = x_f - x$ and the output is defined as $t_{go} = T - t$ in every sample point.

Under certain initial conditions, several trajectories are simulated by setting different expected impact-angles. Specific simulation conditions are shown in **Table 1** From the above, it can be seen that there are 16 trajectories simulated in total.

3.3. Training Result Analysis

Neural Net Fitting toolbox in MATLAB is selected for the neural network training and millions of samples are used to train and test the neural network model. **Figure 5** shows the training process of the neural network. Mean squared error (MSE) is used as the index to evaluate the network performance. When the training epochs are 760, the prediction error is less than 10^{-3} which is sufficient to achieve our goal and the trend of the curves has been decreasing.

4. Simulation Results and Discussion

4.1. Simulation Results

The simulated model selects the actual missile model shown in [18]. Initial conditions are shown as **Table 1**. The simulation ends if the distance between the missile and the target is less than $0.1m$.

Firstly, the effectiveness of the guidance law proposed is verified. Suppose the expected impact-angle is $-115deg$ which is not within the dataset of neural net-

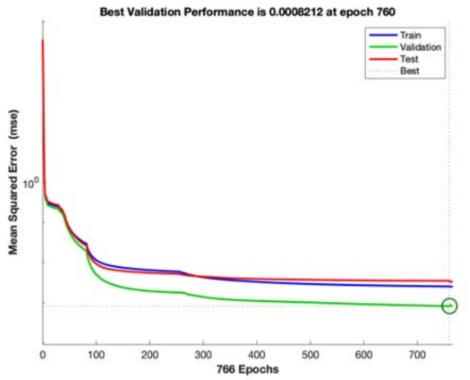

Fig. 5. Training process of the nerual network

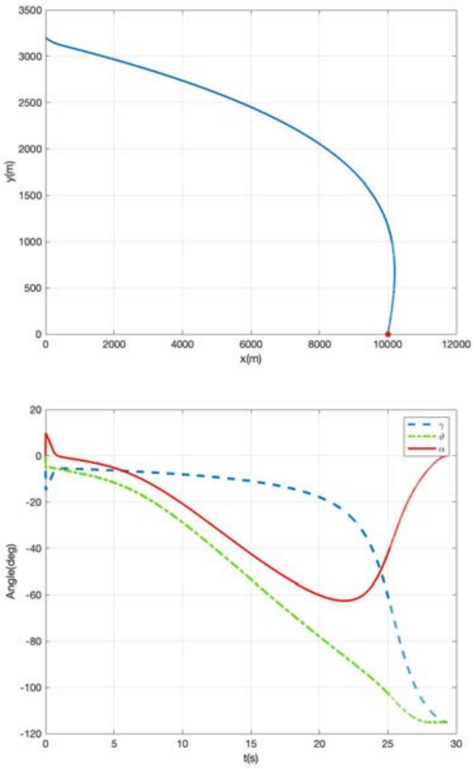

Fig. 6. Engagement results

work training. Simulation results are shown in **Fig. 6**. It can be seen that, on the premise of hitting the target accurately, the guidance law proposed can not only constrain the impact-angle effectively, but also make the AoA converge to zero at the time of the strike, that is, make the nose of the missile point in the direction of the velocity vector.

Next, simulation is carried out for different expected impact-angles namely, $-5deg$, $-45deg$, $-90deg$ and $-135deg$ and the corresponding results are shown in **Fig. 7**. It can be seen that the proposed guidance law can accurately hit the target under different expected constraints and terminal zero AoA is guaranteed.

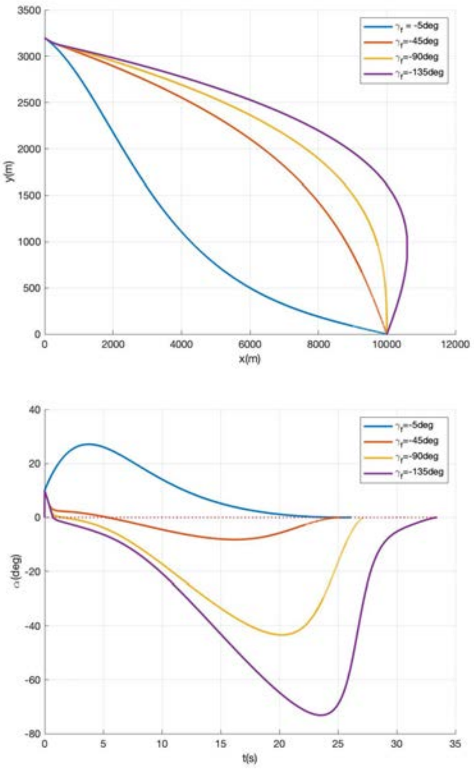

Fig. 7. Engagement results with different impact angles

Finally, the proposed guidance law is compared with the guidance law clarified in [16]. Suppose the expected impact-angle is set as $-45deg$. Under the same conditions, simulation results are shown in **Fig. 8**. For ballistic simulation time under MATLAB 2020a, the proposed guidance law spends $0.63s$ while the other guidance law spends $6.69s$ on average, which is because the fourth-order SDRE needs to be solved online using the guidance law in [16] but the guidance law designed in this paper is reduced to two second-order SDREs by using neural network and the analytical solution is obtained directly. Besides, the curved degree of the trajectory using the proposed guidance law is flatter than the other one and the AoA using the proposed guidance law converges to zero more quickly in the later stage.

4.2. Discussion

1. The operation time of the guidance law designed in this paper is far less than that designed in [16], but the former requires offline training in advance.

2. Compared with the guidance law designed in [16], the trajectory of the proposed guidance law is less curved and there is no pull-up process, which can eliminate more interferences.

3. The guidance law proposed in this paper adjusts the AoA later than the guidance law designed in [16], but the convergence speed is faster and the whole flight time is shorter.

4. The approach in this paper is still inadequate. The

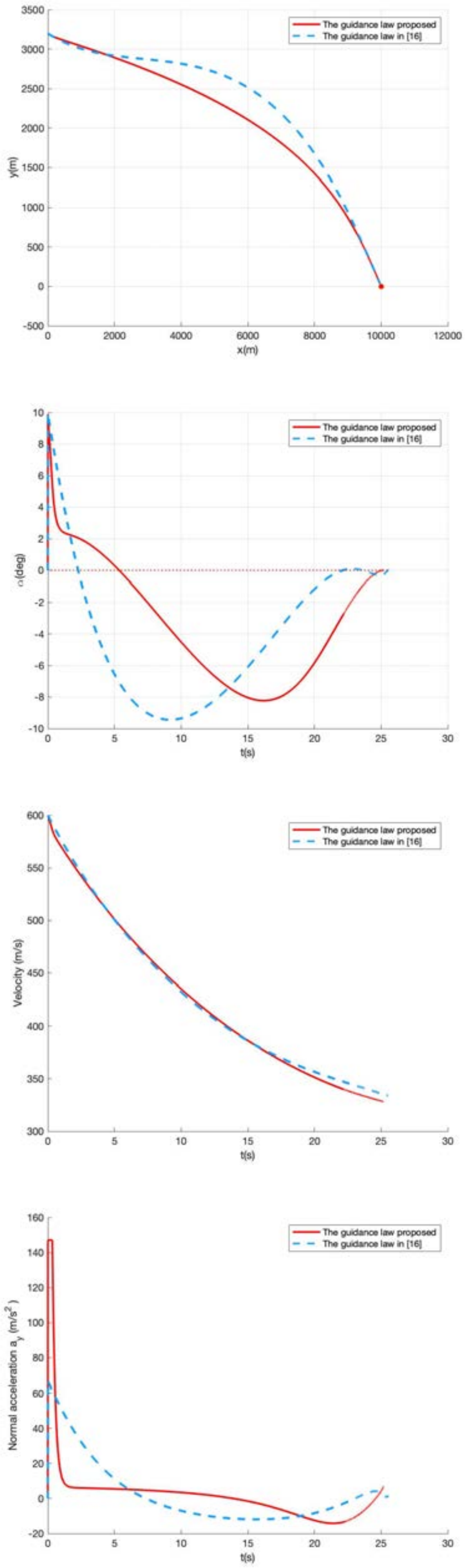

Fig. 8. Engagement results with different expected impact-angles

velocity of the guidance law proposed is slightly lower at the time of the strike. In addition, it will have a transient large overload that we don't really want to see at the initial stage.

5. Conclusions and Future Scope

In this paper, a pitch controlled impact-angle-constrained and the terminal AoA constrained guidance law has been developed using the SDRE algorithm and the neural network algorithm. By fitting the time-to-go through the neural network, the fourth-order SDRE is directly reduced to two second-order SDREs by implementing the measurable flight states of the missile (especially the altitude information) in real-time, and the impact-angle and the terminal attitude are constrained, so as to realize the zero AoA hitting and greatly improve the destructivity of the strike.

In the future, this approach can be applied to more complicated models, such as models with roll and yaw, nonlinear and strongly coupled hypersonic vehicles, models that take into account terrestrial and climatic conditions, and so on. Also, the neural network can be combined with more constrained guidance problems, so that it can give full play to its intelligence advantage in guidance and control field.

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